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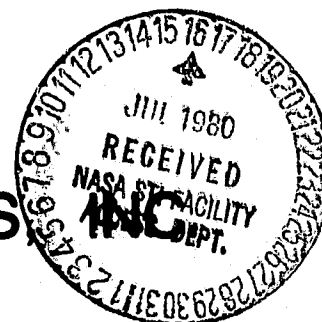
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MISSION ANALYSIS DATA FOR INCLINED GEOSYNCHRONOUS ORBITS PART 1

ANALYTICAL AND
COMPUTATIONAL
MATHEMATICS



MISSION ANALYSIS DATA FOR INCLINED
GEOSYNCHRONOUS ORBITS

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MISSION ANALYSIS DATA FOR INCLINED
GEOSYNCHRONOUS ORBITS

| SECTION | PAGE |
|--|------|
| 1.0 Introduction | 3 |
| 1.1 Background | |
| 1.2 Overview of this Report | |
| 1.3 Applications of Inclined Geosynchronous Orbits | |
| 1.4 The "Halo" Orbit Concept | |
| 1.5 Summary of Results | |
| 2.0 Orbit Geometry and Definitions | 10 |
| 2.1 Orbital Elements and Coordinate Systems | |
| 2.2 Classification of Geosynchronous Orbits | |
| 3.0 Perturbing Effects on Geosynchronous Orbits | 20 |
| 3.1 Ellipticity of the Equator | |
| 3.2 Sun, Moon and Earth Oblateness | |
| 3.3 Solar Radiation Pressure | |
| 4.0 Time Histories of Orbital Elements | 24 |
| 4.1 Earth-Relative Mean Longitude | |
| 4.2 Motion of the Orbit Plane | |
| 4.3 Change in the Eccentricity | |
| 5.0 Velocity Requirements for Orbit Maintenance | 66 |
| 5.1 In-Plane Delta V | |
| 5.1.1 Mathematical Developments | |
| 5.1.2 Numerical Results | |
| 5.2 Out-of-Plane Delta V | |
| 5.2.1 Mathematical Developments | |
| 5.2.2 Numerical Results | |
| 6.0 Discussion of Multisatellite Systems | 78 |
| 6.1 Problem Areas | |
| 6.2 Mission Design Concepts | |
| References | 80 |
| Appendix A: Bibliography | |
| Appendix B: Listings of Computer Programs | |

MISSION ANALYSIS DATA FOR INCLINED GEOSYNCHRONOUS ORBITS

by

Otis F. Graf, Jr. and K.C. Wang

1.0 INTRODUCTION

1.1 Background

The geostationary equatorial orbit (GEO) has been extensively used during the past 10-15 years. Satellites in this orbit appear nearly stationary above the earth and are ideal for applications in communications and earth observations.

With the increasing applications of the GEO, the available space in the orbital arc is now being depleted. For example, the NASA Space Transportation System is expected to launch approximately 160 geostationary missions during the next ten years (Reference 1). The satellites need to be separated by a few degrees in order to avoid physical and/or radio frequency interference.

Because of the extensive planned geostationary mission traffic and the limited number of slots available, international organizations are attempting to gain authority over slot allocations. The U.S. has no inherent claim to geostationary "real estate" since it has no territory over the equator. The geostationary arc of interest to the domestic U.S. (55 deg W. longitude to 135 deg W. longitude) may in the future be shared

with South American countries. Also, certain futuristic projects (such as the solar power satellites and space manufacturing) will require large amounts of the geosynchronous or geostationary arc.

For the reasons cited above, it will be important in the future to pack more satellites into useful geosynchronous orbits. A promising proposal that has been put forth is to use geosynchronous orbits whose planes are inclined to the equator. A satellite in such an orbit has a ground track that is a figure eight. If several satellites are placed in orbits that have the same figure eight ground track, they will all cross the equator at the same longitude, called the "gateway". The gateway may require only a few degrees of the geostationary arc, while accommodating possibly 10-12 satellites in inclined orbits. As the satellites pass through the upper (or lower) latitudes, their ground track would appear to be nearly circular about a point on the earth's surface. From this point of view, they are called "halo" orbits.

1.2 Overview of this Report

Most of the earlier studies of inclined geosynchronous orbits have concerned geometrical relationships with idealized (unperturbed) orbits. In practice, however, the orbits will be perturbed by gravitational and non-gravitational forces. The result is that the orbits will be altered, and corrective maneuvers must be executed by on-board control systems. Principal orbit perturbations are due to earth oblateness, earth equatorial ellipticity, sun and moon gravity, and solar radiation pressure.

For equatorial geosynchronous (geostationary) orbits, the effects of these perturbations are well known. However, for inclined geosynchronous orbits, less information is known about the orbit perturbations or the orbit maintenance delta-V requirements.

The purpose of this report, therefore, is to provide certain data that is needed for preliminary design of inclined geosynchronous missions. In particular, the objectives are to:

- Study the perturbations on circular orbits with inclinations up to 60 degrees.
- Determine the time histories of the orbital elements for periods of up to ten years.
- Determine the velocity management requirements needed to cancel the major perturbing effects.
- Present a compilation of data on inclined circular geosynchronous orbit characteristics.

Section 1 of this report introduces the problem and gives the scope and purpose of the study. A summary of the principle results is also given.

Section 2 describes the inertial and earth-fixed (rotating) coordinate systems, as well as orbit parameters and elements. The complete family of geosynchronous orbits is discussed. It is shown that circular, inclined geosynchronous orbits comprise only one set in this family.

Section 3 gives a discussion of the major orbit perturbations, and their separate effects on the geosynchronous orbit.

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Section 4 gives detailed information on the orbit perturbations of inclined, circular geosynchronous orbits. The major emphasis is to provide time history data of certain orbital elements.

Section 5 provides a determination of orbit maintenance delta velocity requirements to counteract the major orbit perturbations. The purpose is to provide order of magnitude estimates, and to show the effects of orbit inclination on delta-V.

Section 6 discusses some of the considerations in mission design for a multisatellite system, i.e. a "halo" orbit constellation.

Section 7 presents some conclusions and recommendations based on this study.

Appendix A gives a bibliography of papers and reports on the subject of geosynchronous orbits. There is a wide body of literature on this subject. The bibliography includes papers of interest to this study, as well as background papers concerned with the fundamentals of orbital mechanics.

Appendix B contains a listing of all the computer programs that were developed for this study. These programs are on the Interdata 8/32 computer at the NASA Johnson Space Center. All routines are thoroughly documented with comment cards.

1.3 Applications of Inclined Geosynchronous Orbits

Several authors have discussed the possible applications of Inclined Geosynchronous Orbits (IGO). Generally these orbits are useful for applications where there must be a long viewing time from the earth. However it is not necessary that the satellite be fixed relative to the surface of the earth. The following applications have been proposed for IGO orbits:

- Earth observation
- Solar Power Satellite
- Communications
- Military applications

Applications and ground tracks are discussed by Akin (Reference 2), Bielkowitz (Reference 3) and Graf (Reference 4).

1.4 The "Halo" Orbit Concept

There is an additional application of IGO that has been proposed by Fielder (Reference 5). His suggestion is to establish a constellation of satellites in inclined circular geosynchronous orbits, such that each satellite in the constellation follows the same figure eight ground track (see Figure 1). At the higher latitudes (which are of interest for the U.S.), the satellites would appear to rotate in a nearly circular orbit relative to the earth, hence the term "Halo" orbits.

The advantage of the "Halo" orbit scheme is that it could significantly increase the number of usable satellites in geosynchronous orbit.

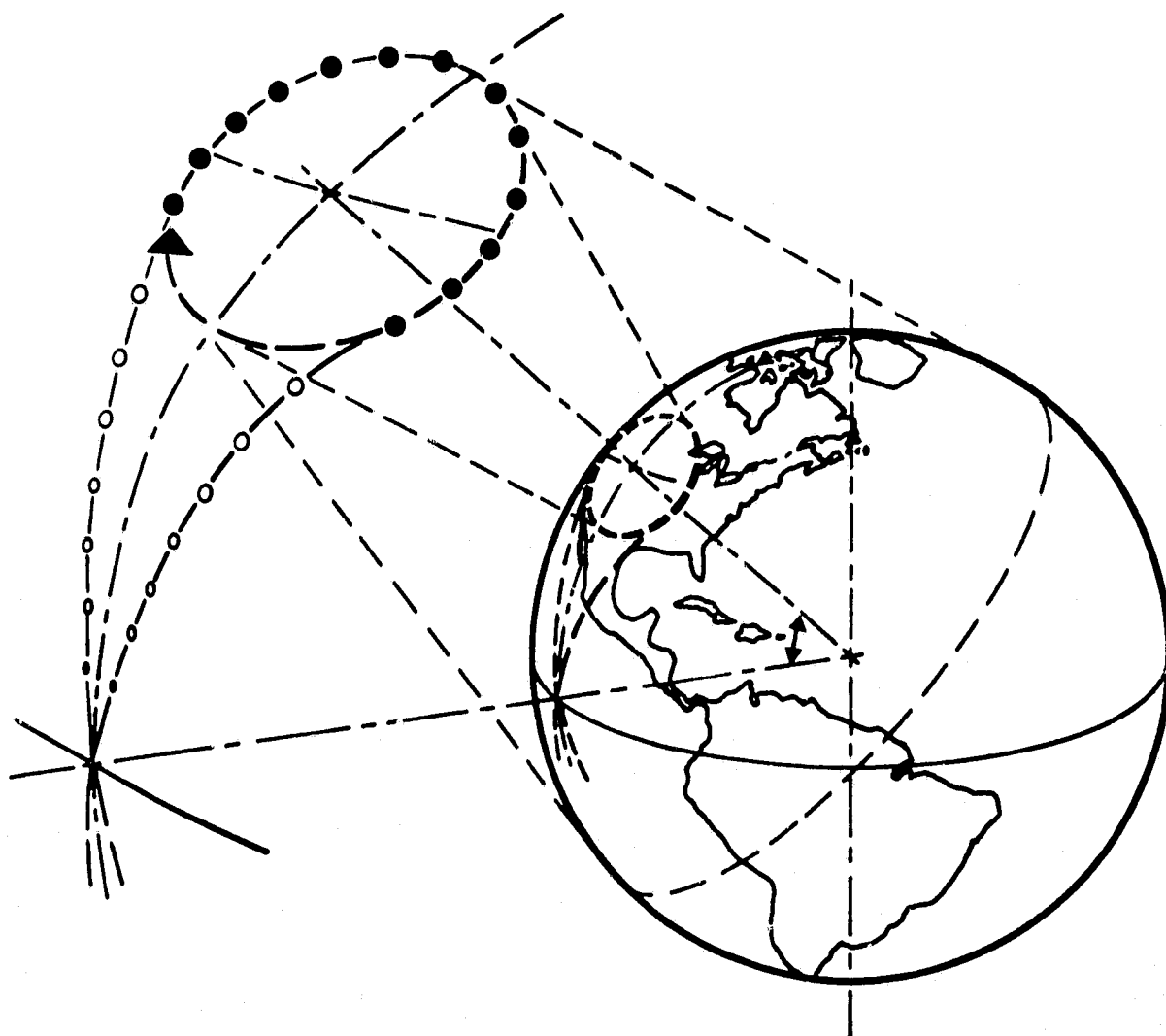


Figure 1- "Halo" Orbit Concept

1.5 Summary of Results

The general results of this study can be summarized as follows:

- The general behavior of the geosynchronous orbit perturbations are similar for the inclined (up to 60 deg.) and equatorial cases. However, the magnitudes of the perturbing accelerations can be different by several orders of magnitude.
- When the orbit is allowed to have a significant inclination, the mission design becomes more complicated. For example, assuming the inclination is given, the inertial orientation of the orbit Ω must still be specified. Additionally, orbit perturbations act in a different way for different values of Ω .
- The drift in mean longitude is generally less for inclined orbits. Likewise, delta-V to cancel this drift will also be less for the more inclined orbits.
- The out-of-plane orbit maintenance delta-V for inclined orbits can be as much as four times that for equatorial orbits. Also, the amount of delta-V required depends on orbit plane orientation relative to inertial space. This means that each satellite in a "Halo" orbit constellation would have unique delta-V requirements.
- By far the most important source of orbit maintenance delta-V is the out-of-plane component. This could be 100 times larger than the in-plane delta-V.

2.0 ORBIT GEOMETRY AND DEFINITIONS

In this section, we take the opportunity to review the geometric relationships between the geosynchronous orbit and coordinate systems. The orbital elements that are used in the following sections will be defined here. Also, this section will categorize the various types of geosynchronous orbits.

2.1 Orbital Elements and Coordinate Systems

Define a rectangular cartesian coordinate system with its origin at the center of the earth. The X-axis lies in the equatorial plane and is directed toward aries (the vernal equinox). The Z-axis lies along the rotational axis of the earth in the direction of the north pole. The Y-axis lies in the equatorial plane and completes the right-handed coordinate system (See Figure 2(A)).

The following orbital parameters are defined in Figures 2(A) and 2(B):

- I : Inclination,
- Ω : Longitude of the ascending node,
- u : Argument of perigee,
- f : True anomaly,
- r : Satellite radial distance from the earth's center,
- a : Semi-major axis,

- r_a : Apogee distance,
- r_p : Perigee distance,
- E : Eccentric anomaly.

We also define the orbital eccentricity as

$$e = \frac{r_a - r_p}{2a} .$$

In order to study the satellite motion relative to the rotating earth, it is useful to introduce polar coordinates and a rotating coordinate system that is fixed in the earth. Refer to Figure 2(C). Let the X' -axis lie in the equatorial plane at the intersection of the Greenwich meridian and the equator. The Z' -axis and the Z -axis coincide. The Y' -axis lies in the equatorial plane and completes the primed coordinate system.

We define the following variables:

- ϕ_c : Angle between the X - and X' -axes, generally referred to as the "Greenwich Hour Angle",
- Λ : The satellite's longitude with respect to the Greenwich meridian,
- θ : The satellite's latitude.

Also, notice that for a satellite in an inclined, circular 24-hour orbit, the longitude Λ will have some mean (or average) value over the course of a day (Fig. 2(D)). The ground track is a figure eight and will cross the equator at a longitude, which is the mean longitude. We refer to mean longitude as λ , a parameter which indicates where the ground track is located at any time. At the instant that the satellite crosses the

equator going north, we have the relation

$$\lambda = \phi_e + \lambda.$$

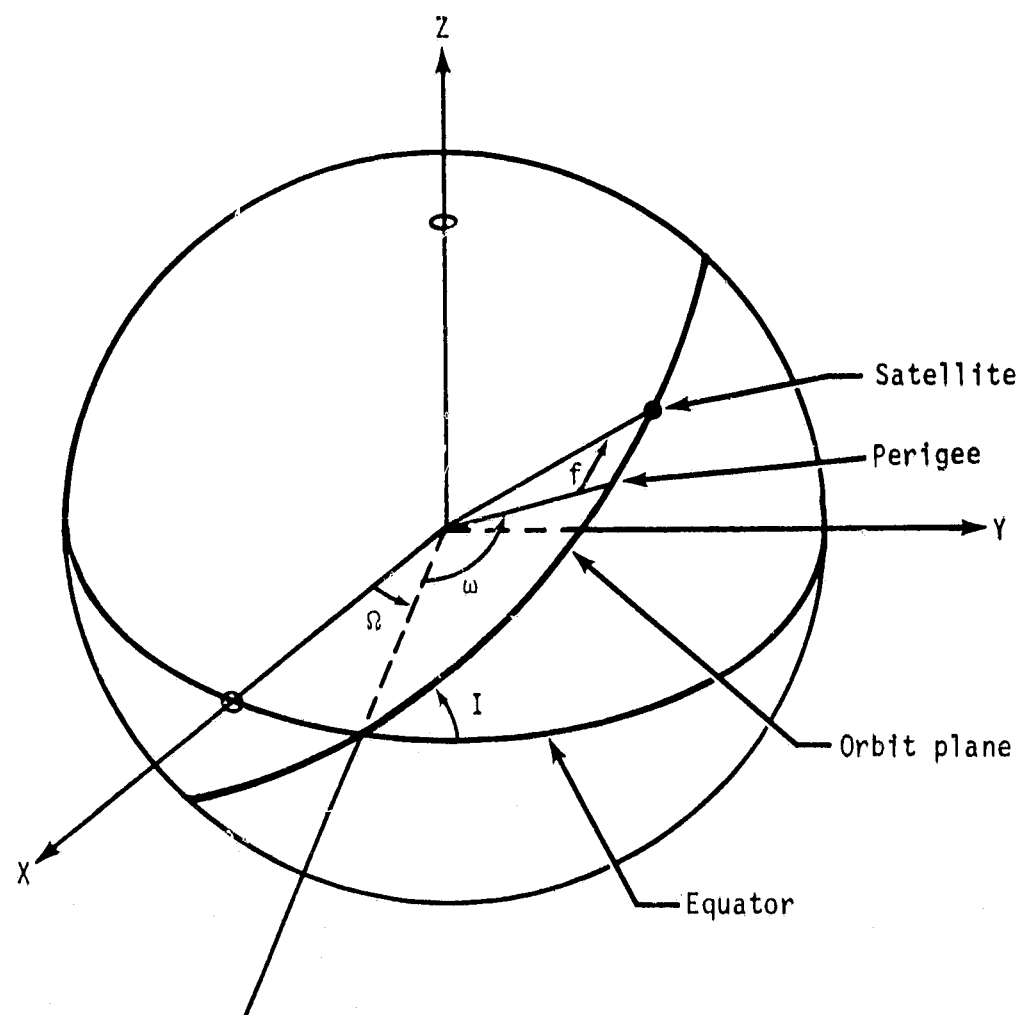
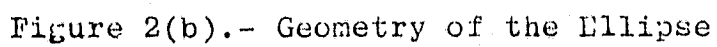


Figure 2(a).- Orbit parameters.



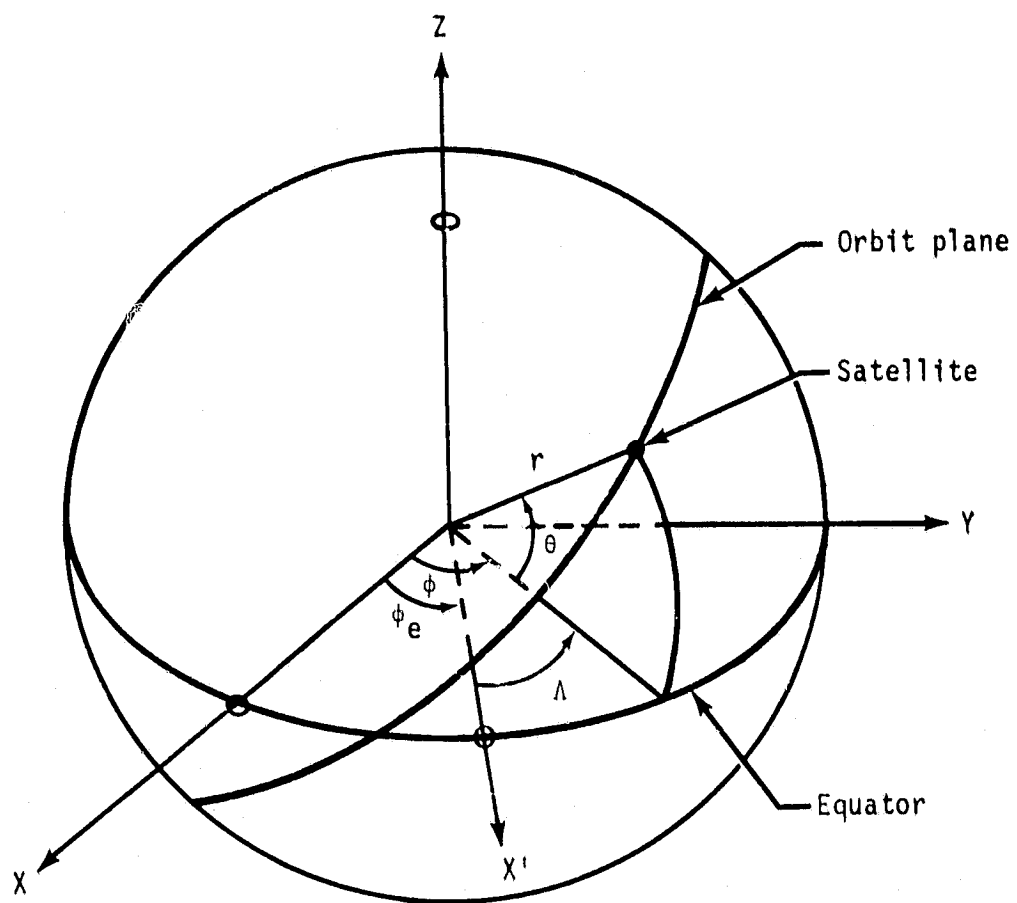


Figure 2(c).- Polar coordinates.

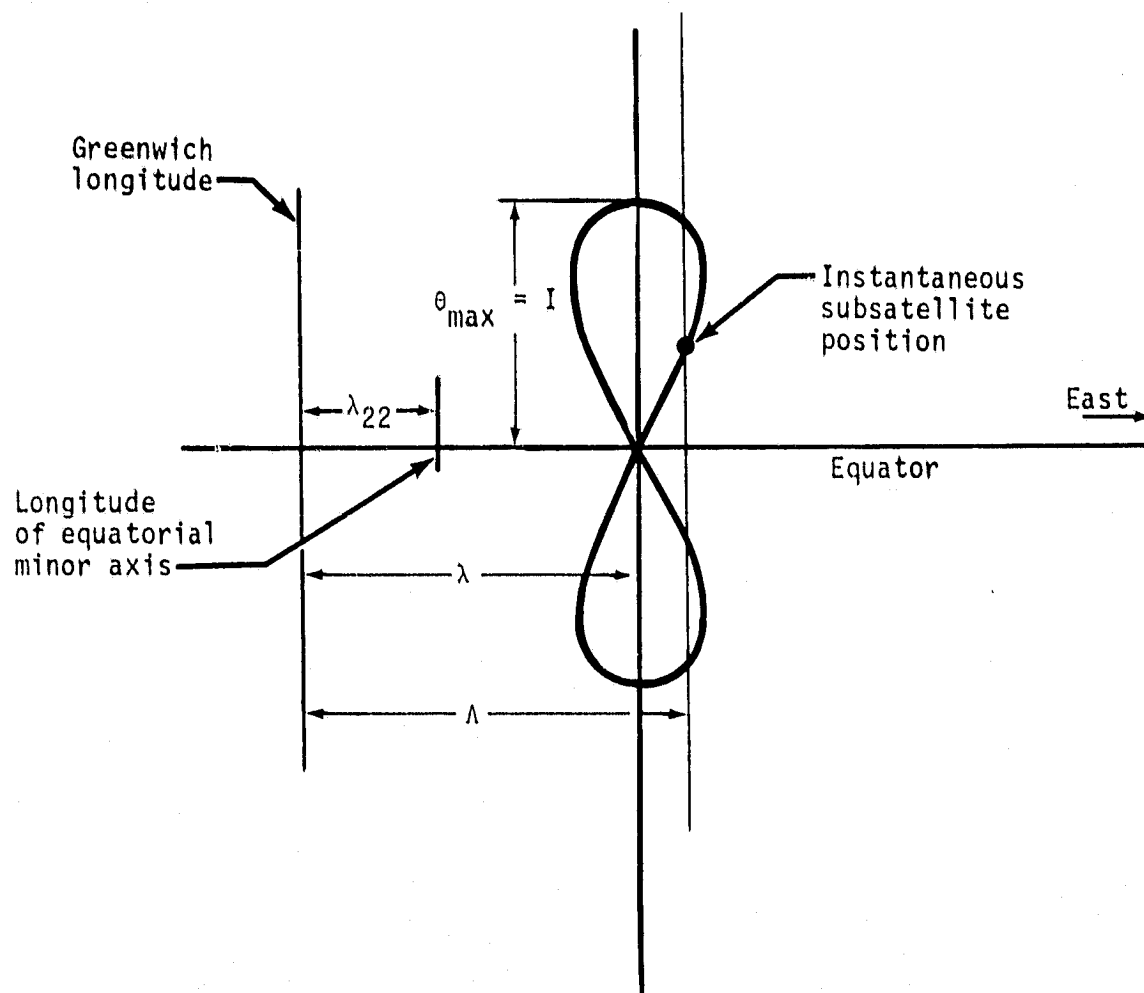


Figure 2(d).- Satellite groundtrack.

2.2 Classification of Geosynchronous Orbits

In this report, the term Geosynchronous Orbit (GO) refers to an orbit that has a period of one sidereal day, i.e. the satellite's orbital period is equal to the period of rotation of the earth relative to inertial space. Thus, a satellite in such an orbit appears to follow the same ground track, day after day. For these orbits, the inclination and eccentricity are not necessarily zero. Therefore, their ground tracks will be closed curves of various shapes. Examples of such ground tracks are given in References 2, 3, 4, 6 and 7.

The geosynchronous orbit is the most general type of orbit that is under consideration here. Indeed, it defines a family of orbits that have the following general characteristics:

- The ground track is a closed curve.
- The ground track is repeated every day.
- The satellite crosses the equator twice each day.

A subset of the general case includes the circular orbits with an arbitrary inclination and longitude of ascending node. These are called "Inclined Circular Geosynchronous Orbits" (ICGO), and are the subject of this report. Their ground tracks are the shape of the familiar figure eight.

We suggest the breakdown of the GO family of orbits into sets and subsets, as shown in Figure 3. Note that the Equatorial Circular Geosynchronous Orbit (ECGO) is generally referred to as the Geostationary Equatorial Orbit (GEO). A Satellite in such an orbit appears to be stationary above the rotating earth. There is only one orbit in this set.

Each ECGO satellite is in the same orbit, but at a different position within the orbit. The satellite's position can be specified by only one parameter, i.e. its longitude.

When an orbit is designed to have a significant inclination and/or eccentricity, the number of degrees of freedom for the orbit are increased. The degrees of freedom for each GO subset are shown in Table 1. Note that the ECGO set has only one degree of freedom, i.e. the mean longitude. The ICGO set has two additional degrees of freedom, i.e. the inclination and ascending node.

The study in this report is concerned with the Inclined Circular set of geosynchronous orbits (ICGO). The following analyses in Sections 4 and 5 assumes that the eccentricity is small and negligible, and that the inclination is in the range $0 < I < 60$.

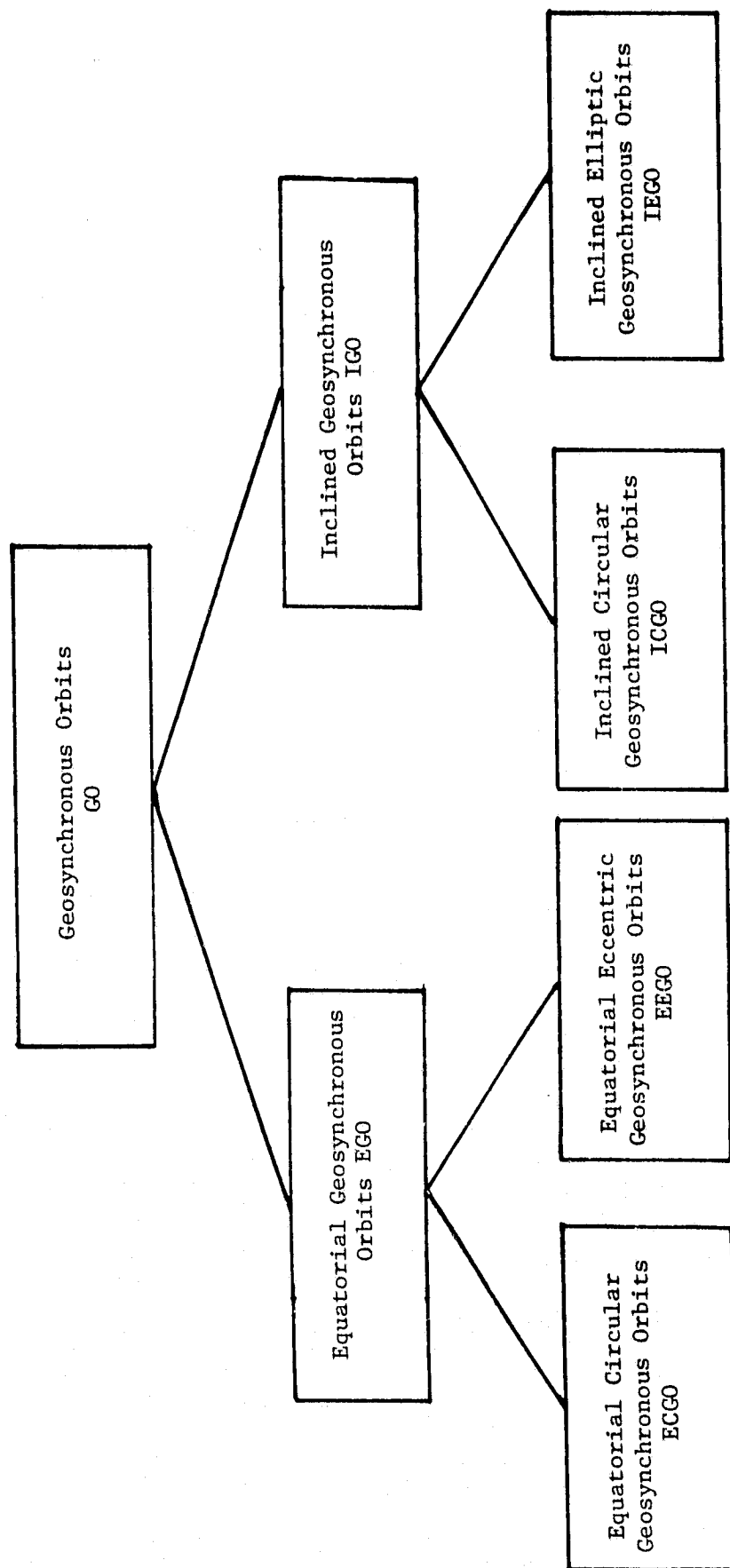


FIGURE 3: Breakdown of the GO Family of Orbits

| ORBIT PARAMETER | | | | | | | |
|-----------------|-------------|-----------|--------------|--------------------------|-------------------------|-------------------------|-----------------------|
| GO Subset | Period T | Ecc. e | Inclin. I | Arg. of Per. ω | Ascnd. Node Ω | Mean Long. λ | Degrees of Freedom |
| ECGO | 24 hrs. | 0 | 0 | N/A | N/A | arb. | 1 |
| EEGO | 24 hrs. | arb. | 0 | arb. | N/A | arb. | 3 |
| ICGO | 24 hrs. | 0 | arb. | N/A | arb. | arb. | 3 |
| IEGO | 24 hrs. | arb. | arb. | arb. | arb. | arb. | 5 |

| Valid Ranges |
|--------------------------|
| $0 < e < 1$ |
| $0 < I < 180^\circ$ |
| $0 < \omega < 360^\circ$ |
| $0 < \Omega < 360^\circ$ |

Table 1: Orbit Degrees of Freedom

3.0 PERTURBING EFFECTS ON GEOSYNCHRONOUS ORBITS

In this section we give a brief summary of the major orbit perturbations on geosynchronous satellites. The perturbations of interest are those of long period that can have an appreciable effect over a period of months or years. Short period perturbations (those that have periods on the order of a few days) will cause small oscillations about the path of the long period trends in the orbital elements. The mathematical methods employed specifically exclude these oscillations in order to determine the more important long period effects.

The dominant perturbing accelerations on the satellite are:

- The triaxial earth, i.e. earth oblateness and equatorial ellipticity.
- Sun and Moon gravitation.
- Solar radiation pressure.

The effects of each perturbation are briefly described in this section. Additional background information is given in Reference 6. The quantitative effects of these perturbations are described in Section 4.

3.1 Ellipticity of the Equator

The earth's equatorial ellipticity causes a slow drift (east or west) in the satellite's mean longitude λ . These perturbations are caused by the J_{22} term in the geopotential.

Let \vec{r} be the satellite's position vector in the X,Y,Z coordinate

system in Figure 2(A). Then acceleration due to the geopotential is

$$\ddot{\vec{r}} = \frac{\partial \vec{V}}{\partial \vec{r}} ,$$

where

$$V = \frac{\mu_e}{r} (1 - V_e).$$

The parameter μ_e is the earth's gravitational constant.

Let R_e be the mean equatorial radius of the earth, and let P_n and P_{nm} be Legendre functions and associated Legendre functions, respectively. Then the earth's geopotential can be expressed as (Reference 8)

$$V_e = \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r} \right)^n P_n(\sin \theta) + \sum_{n=2}^{\infty} \sum_{m=1}^n J_{nm} \left(\frac{R_e}{r} \right)^n P_{nm}(\sin \theta) \cos(\Lambda - \lambda_{nm}).$$

For a triaxial earth, the geopotential becomes

$$V_{eT} = \frac{1}{2} J_2 \left(\frac{R_e}{r} \right)^2 (3 \sin^2 \theta - 1) + J_{22} \left(\frac{R_e}{r} \right)^2 \cos^2 \theta \cos 2(\Lambda - \lambda_{22}).$$

Observe that the J_2 term (oblateness) depends on the satellite's altitude and latitude. The J_{22} term (equatorial ellipticity) depends, additionally, on the satellite's longitude relative to the earth reference longitude λ_{22} .

The geopotential parameter λ_{22} is the angle between the equatorial ellipse minor axis and the Greenwich meridian, as shown in Figure 4. The non-dimensional coefficients have the values (Reference 8)

$$J_2 = 1.08248 \times 10^{-3}, \quad J_{22} = 1.77116 \times 10^{-6}.$$

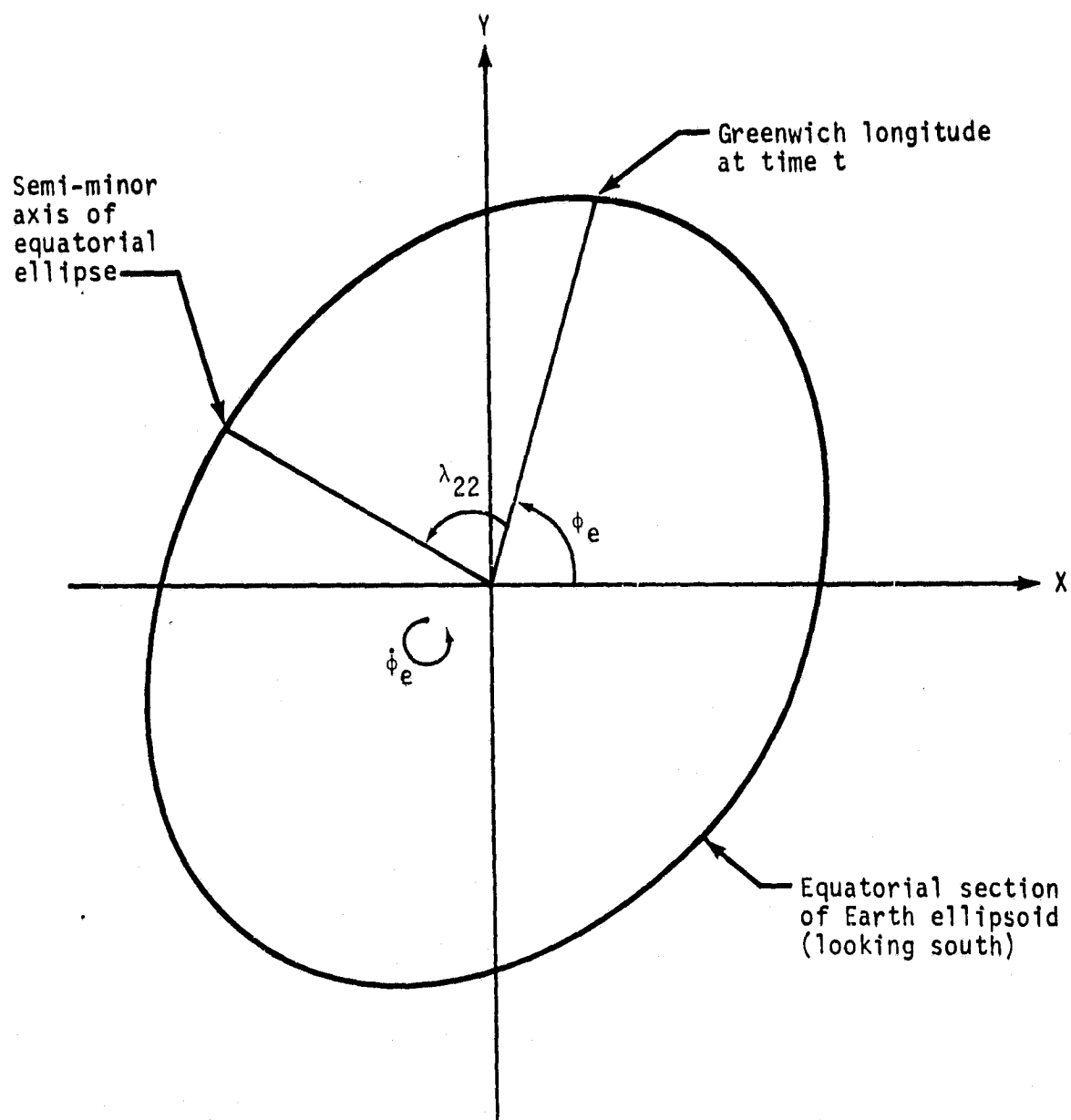


Figure 4.- The Earth's equatorial ellipticity.

3.2 Sun, Moon and Earth Oblateness

The J_2 term in the geopotential combines with sun and moon gravity to cause a long period perturbation of the satellite's orbital plane. The period is more than 50 years.

For I initially zero, the period is 53 years and the amplitude is 15 deg. This means that after 26 years, the inclination will have increased to about 15 deg.

Section 4.2 describes the motion of inclination for arbitrary initial inclinations.

3.3 Solar Radiation Pressure

This non-gravitational perturbation is usually small, but could be appreciable for geosynchronous satellites with large solar collectors. For the Solar Power Satellite (SPS), this perturbation has the order of magnitude of gravitational terms due to sun, moon and J_2 (Reference 4). The solar radiation pressure acceleration is proportional to the satellite's area-to-weight ratio. The major effects of this perturbation is a yearly oscillation of the eccentricity and a rotation of the line of apsides. Section 4.3 gives orbital element time history data for this perturbation.

4.0 TIME HISTORIES OF ORBITAL ELEMENTS

In this section we study the time histories of the orbital elements that are important for geosynchronous missions. We generally rely on analytical formulas to generate the time histories. This allows us to parameterize a broad spectrum of initial conditions. Also, the formulas will be used in Section 5.0 to determine orbit maintenance ΔV .

There are some limitations in the analytical formulas that will be used. For example, each set of perturbations discussed in Section 3 is taken separately. The coupling (i.e. interaction) of the perturbations are ignored since the coupling will be of second order in magnitude compared to the principle effects. However, these formulas should provide sufficient accuracy for mission planning type studies.

In Section 4.1 the long term effects of earth's equatorial ellipticity (J_{22}) are discussed. Section 4.2 discusses the out-of-plane perturbations caused by sun, moon and earth oblateness (J_2). Finally, Section 4.3 describes the perturbations on the orbit's eccentricity due to solar radiation pressure.

4.1 Earth-Relative Mean Longitude

A good description of the longitude drift for equatorial satellites is given by Blitzer in Reference 9. In this section, however, we will use the equations from Wagner (Reference 10) since his equations are valid for non-zero inclinations.

Let λ be the mean daily longitude of the satellite (Section 3.1). This is also the longitude of the satellite as it crosses the equator in its figure 8 ground track. From Reference 10, we have the acceleration of the mean longitude given by the equation

$$\ddot{\lambda} = - \frac{3}{r_s} \left[F_{22} F(I)_{22} \sin 2(\lambda - \lambda_{22}) \right], \quad (4.1)$$

where

$$F_{22} = \frac{6 \mu_e}{r_s^2} \left(\frac{R_e}{r_s} \right)^2 J_{22},$$

$$F(I)_{22} = \frac{(1 + \cos I)^2}{4}.$$

We define the following physical quantities:

μ_e : Gravitational parameter of the earth,

r_s : Radius magnitude of the geosynchronous orbit,

R_e : Radius of the earth,

J_{22} : Geopotential constant,

λ_{22} : Geopotential constant.

The numerical values for these quantities are given in Appendix B.

The function $F(I)_{22}$ is called the "inclination factor". It has the effect of decreasing the acceleration of λ . Thus, it can be seen

from equation (4.1) that the nature of the longitude drift for inclined orbits is similar to the equatorial case.

Notice that only the ellipticity of the equator (i.e. J_{22} term) is included in this analysis. All the geopotential terms are given by Warner in equation (1) of Reference 10. However, we only include the J_{22} term here because that will give sufficient accuracy for our own purposes, and also will demonstrate the nature of the motion.

In using equation (4.1) to describe the motion of the mean longitude, we have made the following assumptions:

- Terms proportional to the eccentricity can be neglected. That is, the orbits are nearly circular.
- The J_{22} term in the geopotential describes the longitude drift with sufficient accuracy.
- There is no coupling of J_{22} with the sun and moon perturbations. This means that the inclination does not change significantly during the time of interest.

The sun and moon have a very small direct effect on the mean longitude drift. Their direct effect is on the order of the neglected geopotential terms. However, in the next section it is shown that the inclination can change by several degrees in a few years. Thus, a "mean" (or average) inclination should be used in equation (4.1). This should give sufficient accuracy for our purposes here.

The long-term drift in mean longitude is shown in Figures 5, 6 and 7. The data presented in these figures was generated from the LDRIFT program that is listed in Appendix B. This program includes the following routines:

LDRIFT: Main program and numerical integration driver. It also prints the data to the CRT screen and writes the data to an output file. This file is later input to the plot program PLOTIT.

LINPUT: Routine for input and initialization of constants and common blocks.

LDERIV: Routine to compute the the right side of the differential equations (4.1).

RKF45 : Runge-Kutta numerical integration routine. This routine is part of the FDS-2 utility library and is not listed in Appendix B.

DISCUSSION OF FIGURE 5

- (1) These figures show the motion in "phase space", i.e. the motion of longitude drift and drift rate. The arrows indicate the direction of the motion.
- (2) Notice that when the drift rate is zero, the drift acceleration is not zero.
- (3) Each geosynchronous satellite will be on one of the closed curves or "orbits" shown in Figure 5. Thus, it is assumed that that a satellite will librate and not be in the "circulation" region.
- (4) The optimum location to place and maintain a satellite is at the position in phase space where the drift rate is zero. That will be on the horizontal line in Figure 5. The satellite will then tend to move away in the direction of the arrows.
- (5) There will be similar plots for mean longitudes in the range 180 to 360 deg.
- (6) The effect of orbital inclination is to squeeze down the the orbits in phase space. The total excursion in longitude is not changed, but the maximum drift rates will be less for the higher inclinations. This is represented as decreased orbit correction delta-V requirements, as shown in Section 5.1

DISCUSSION OF FIGURE 6

- (1) These figures show mean longitude as a function of time, for various initial longitudes and inclinations.
- (2) Notice that the amplitudes of the oscillations depend very strongly on the initial longitude. The stable longitude is near 75 deg. Thus the amplitudes in Figures 6(C) and 6(D) are the smallest of those shown. For $\lambda_0 = 75^\circ$ (or 105° west), the figure would show nearly a straight line.

- (3) The orbit is initialized near the unstable point in Figure 6(G). It is seen that the amplitude is very large, but that the motion is initially very slow. Thus we can expect very small fuel requirements in order to maintain the satellite's position near the unstable points (-15 and 165 deg).
- (4) The effect of an inclination is to reduce the rate of change of the mean longitude.

DISCUSSION OF FIGURE 7

This figure demonstrates how the period of the libration will change as a function of inclination and initial longitude. Notice that the function is highly non-linear. Orbits with larger inclinations have a longer period of libration.

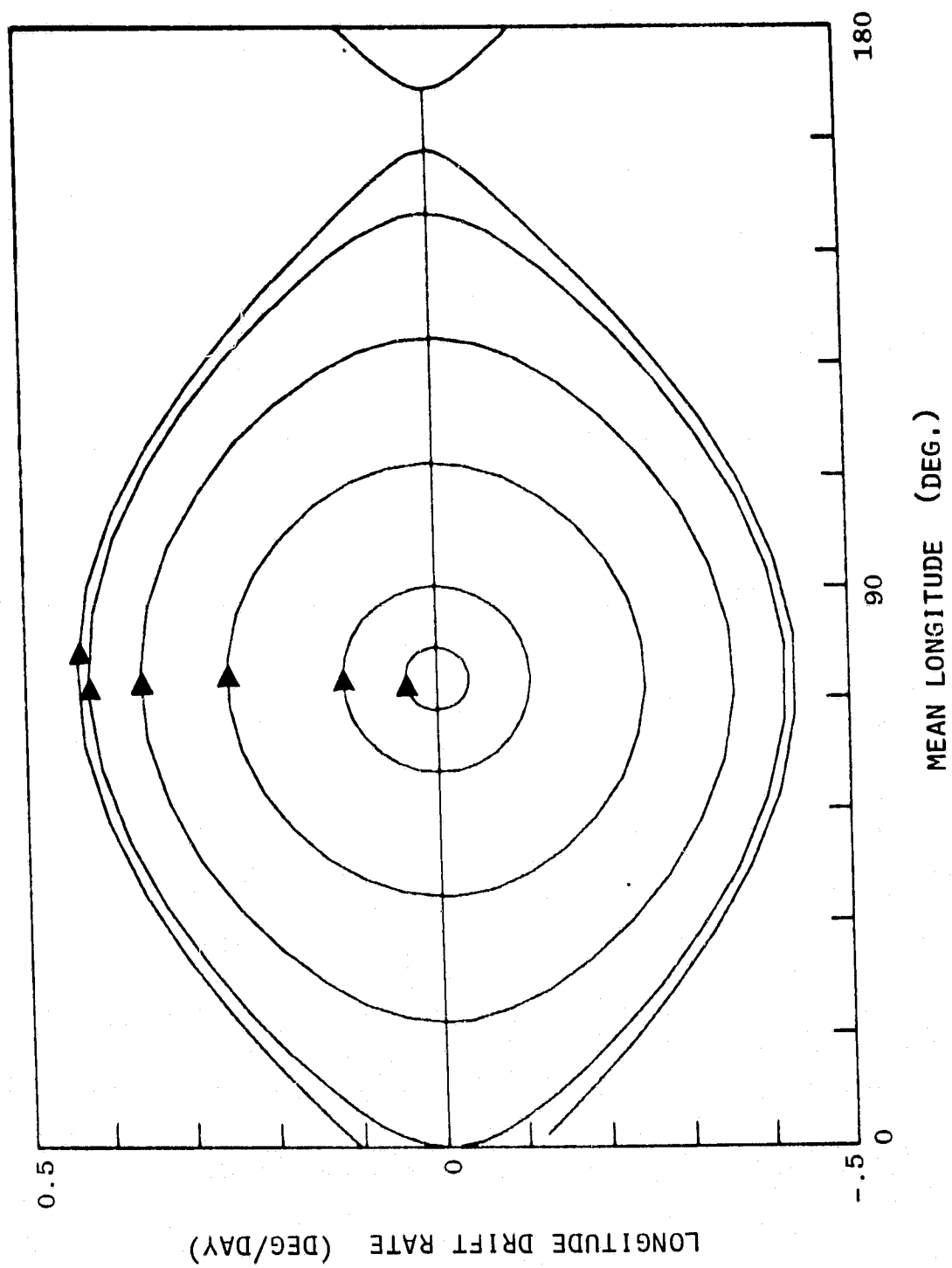


FIGURE 5: DRIFT RATE VS. MEAN LONGITUDE
(A) INCLINATION = 0 DEG.

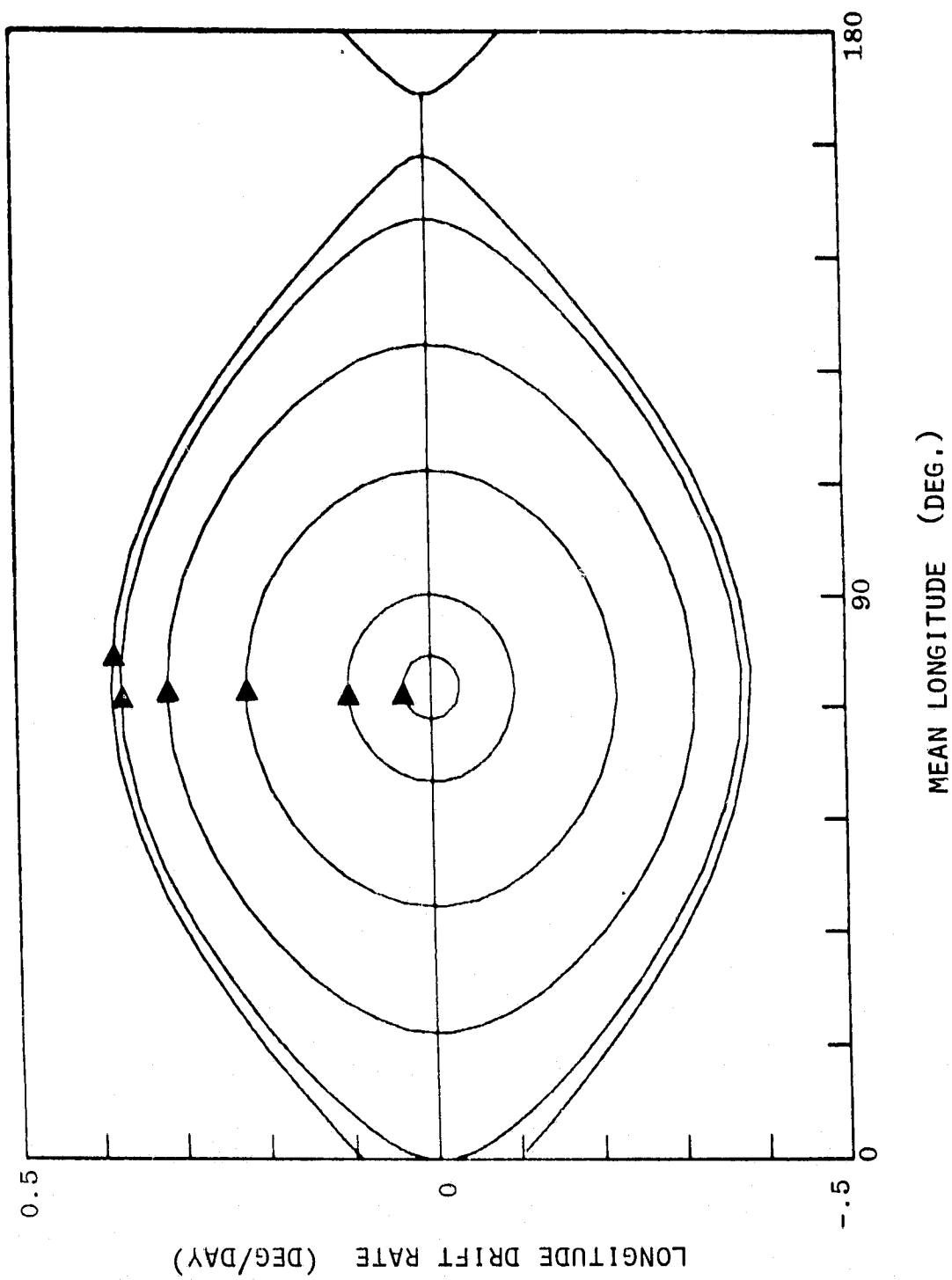


FIGURE 5: DRIFT RATE VS. MEAN LONGITUDE
(B) INCLINATION = 40°

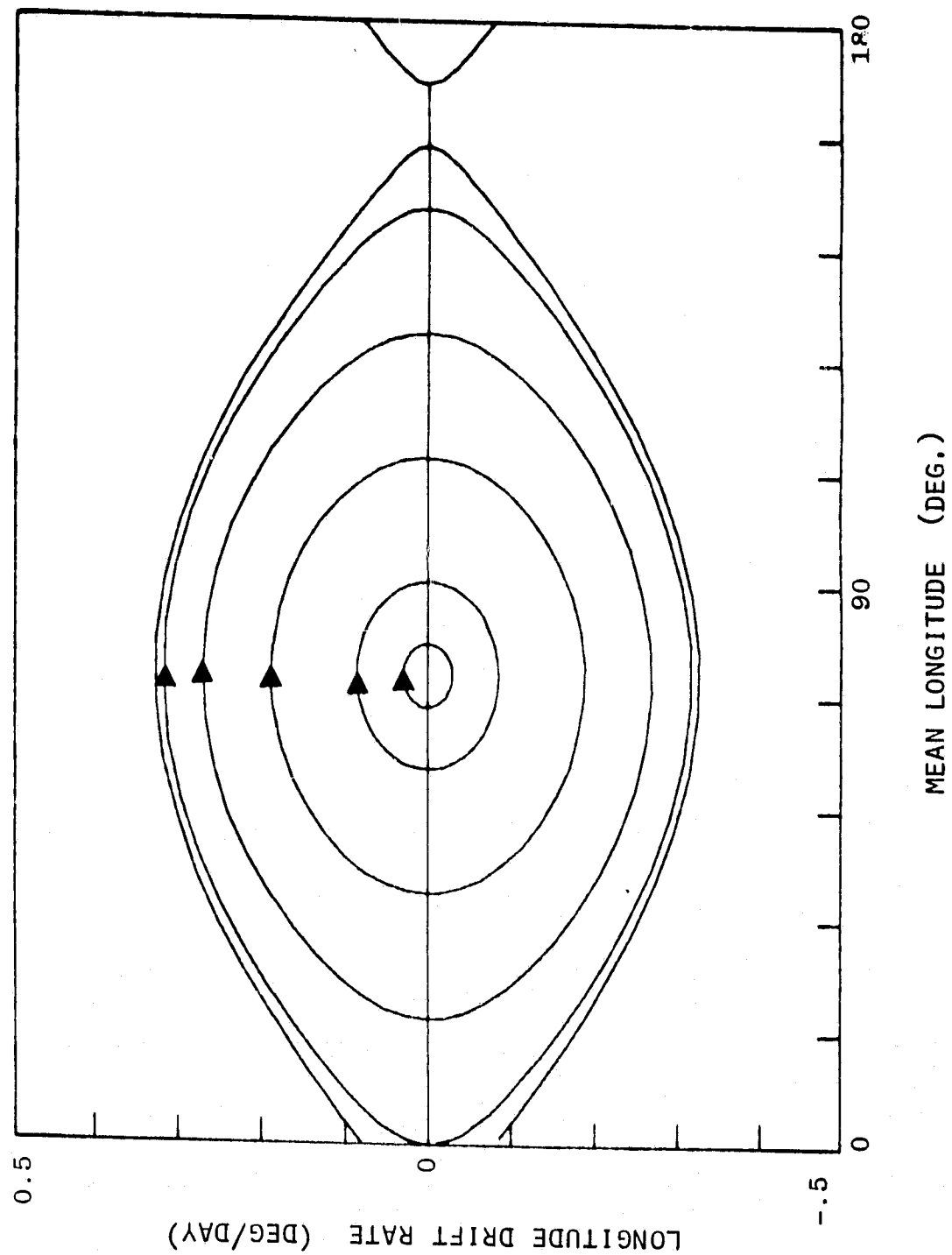


FIGURE 5: DRIFT RATE VS. MEAN LONGITUDE
(C) INCLINATION = 60°

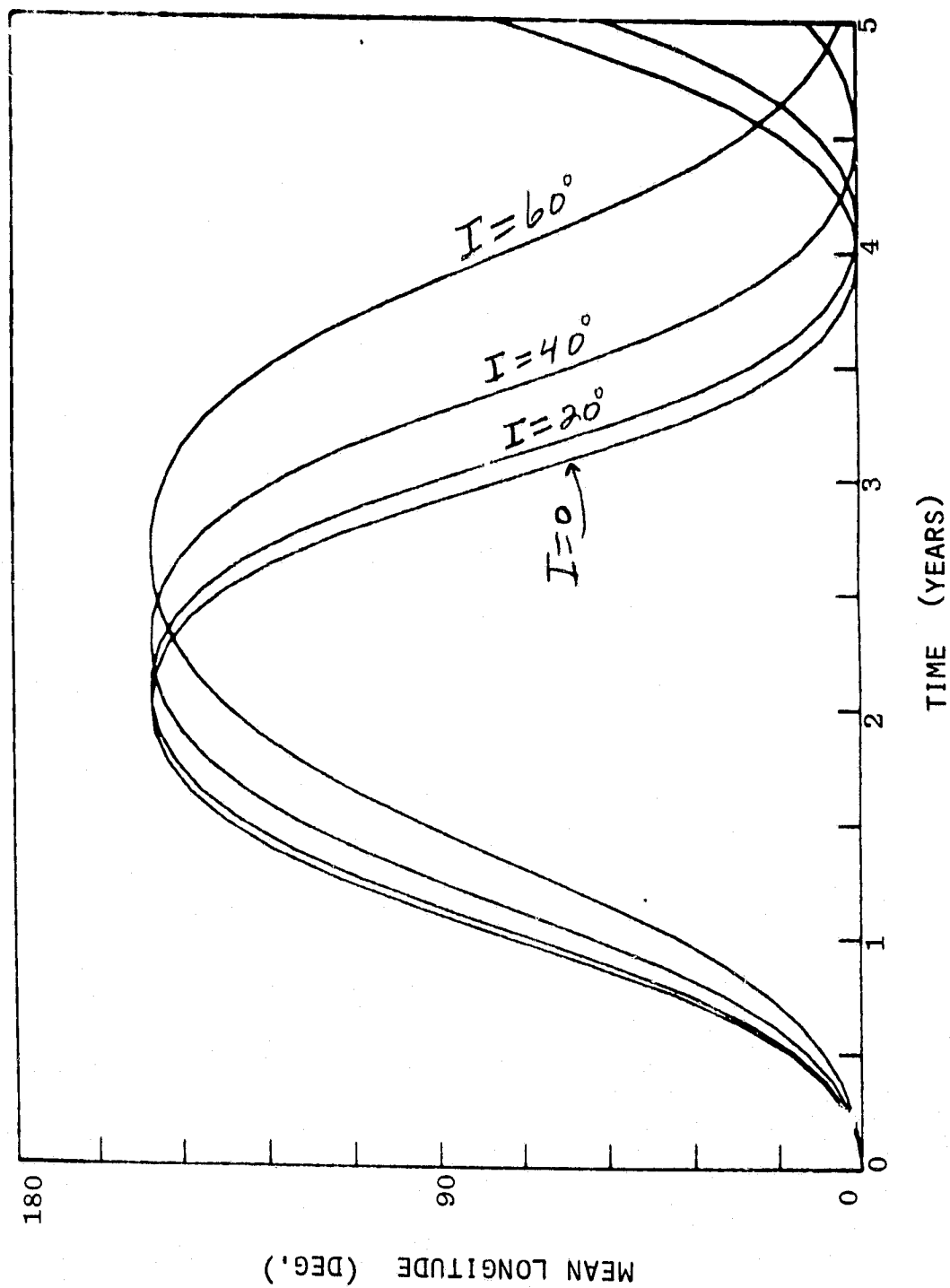


FIGURE 6: MEAN LONGITUDE VS. TIME
(A) INITIAL LONGITUDE = 0 DEG.

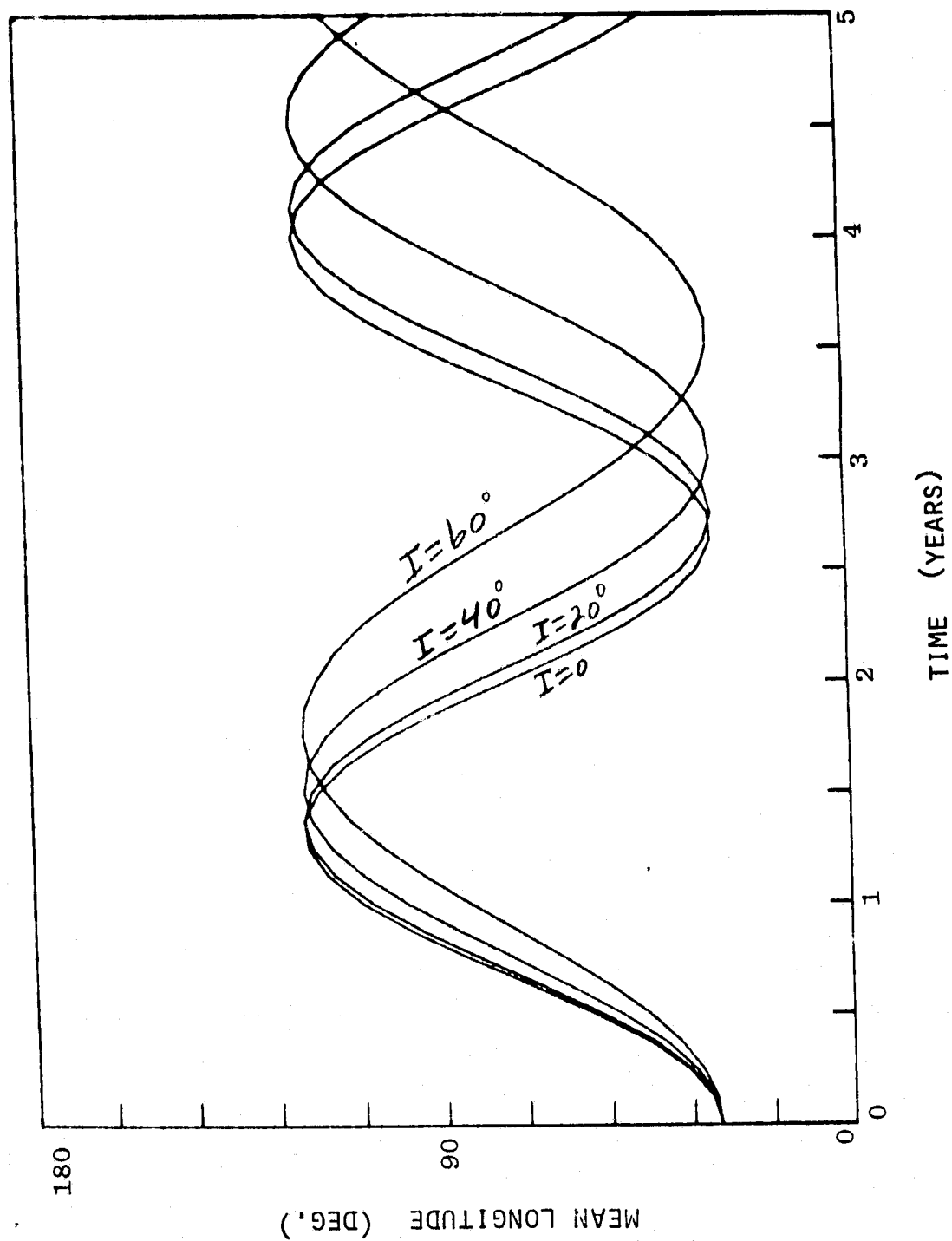


FIGURE 6: MEAN LONGITUDE VS. TIME
(B) INITIAL LONGITUDE = 30°

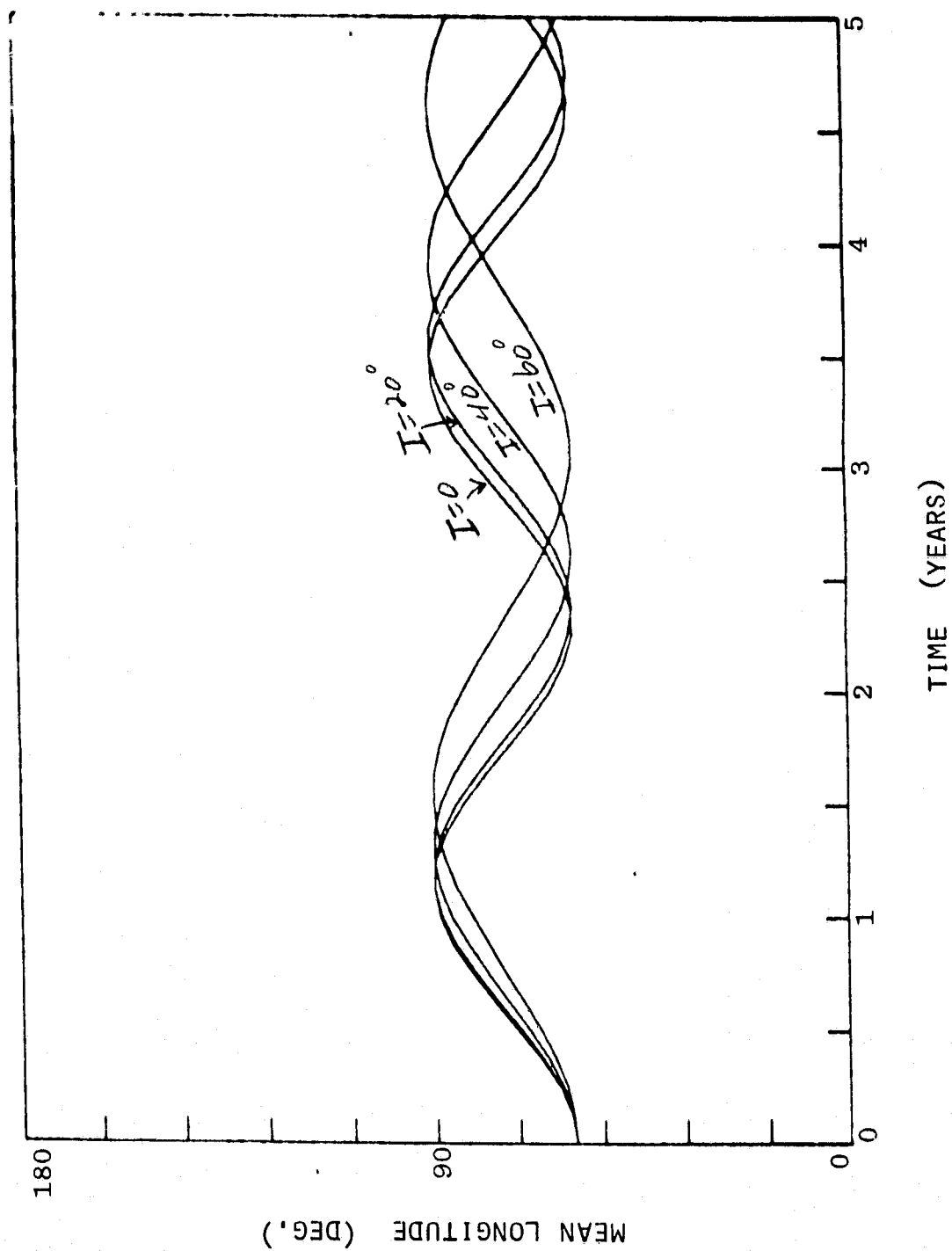


FIGURE 6: MEAN LONGITUDE VS. TIME
(C) INITIAL LONGITUDE = 60°

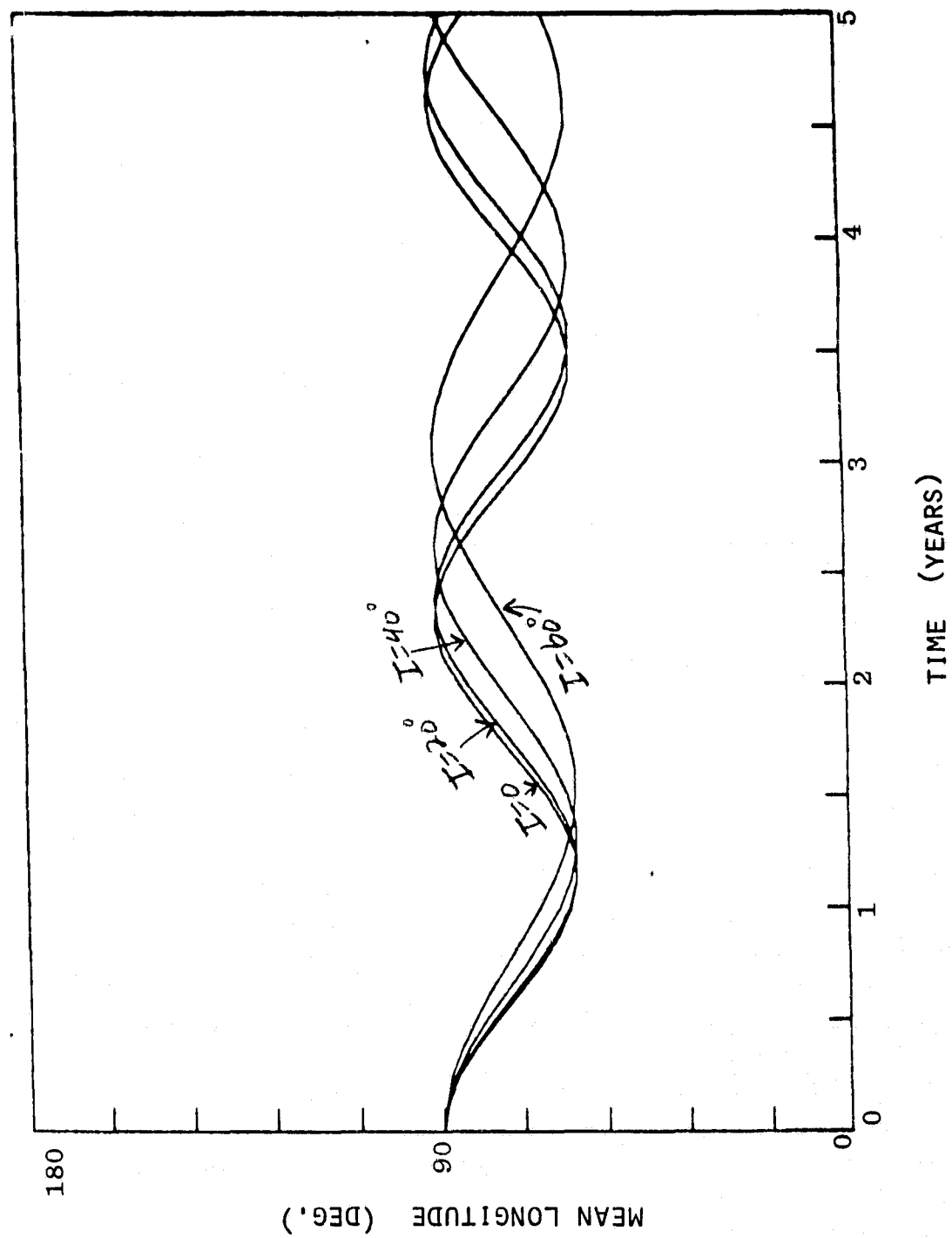


FIGURE 6: MEAN LONGITUDE VS. TIME
(D) INITIAL LONGITUDE = 90°

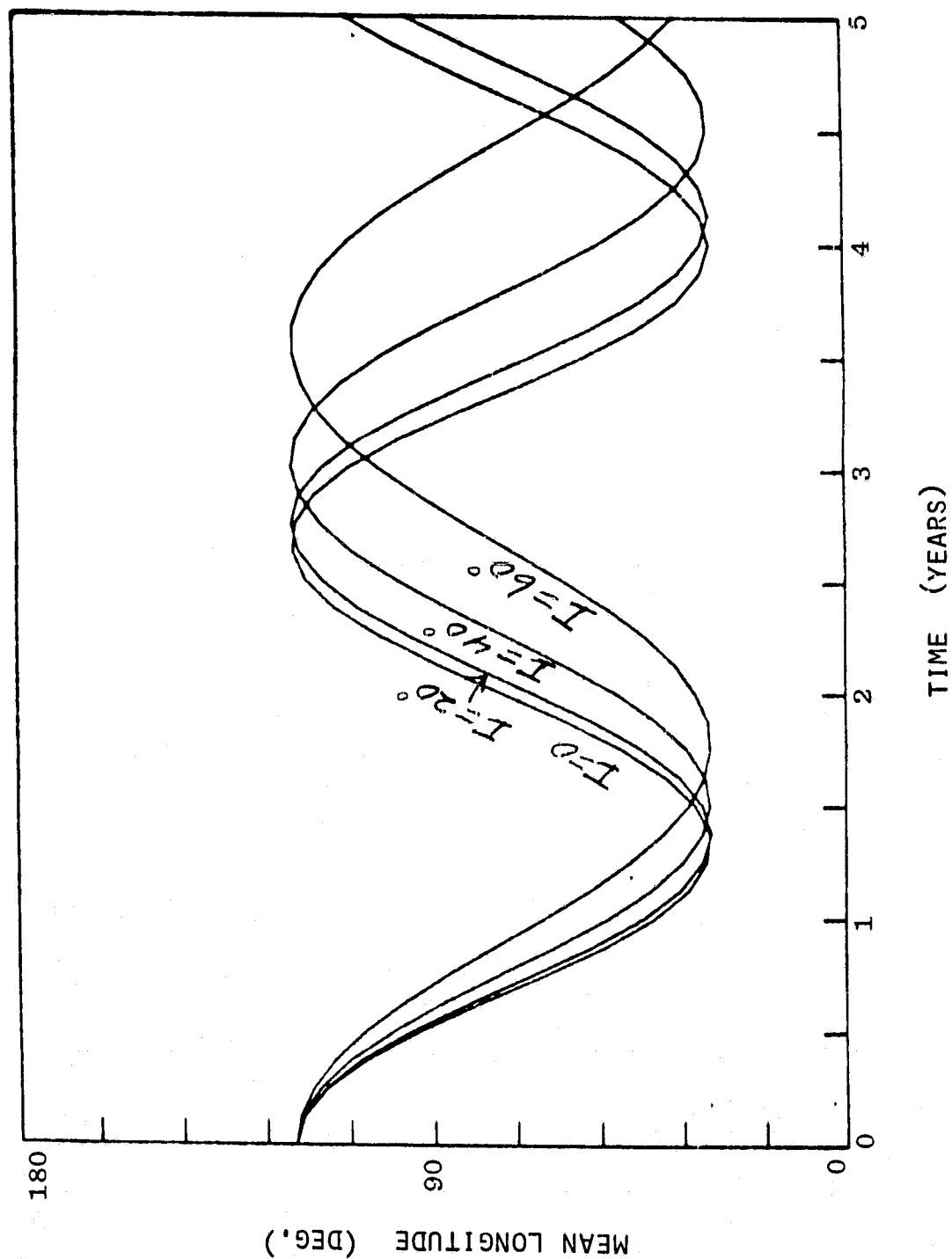


FIGURE 6: MEAN LONGITUDE VS. TIME
(E) INITIAL LONGITUDE = 120°

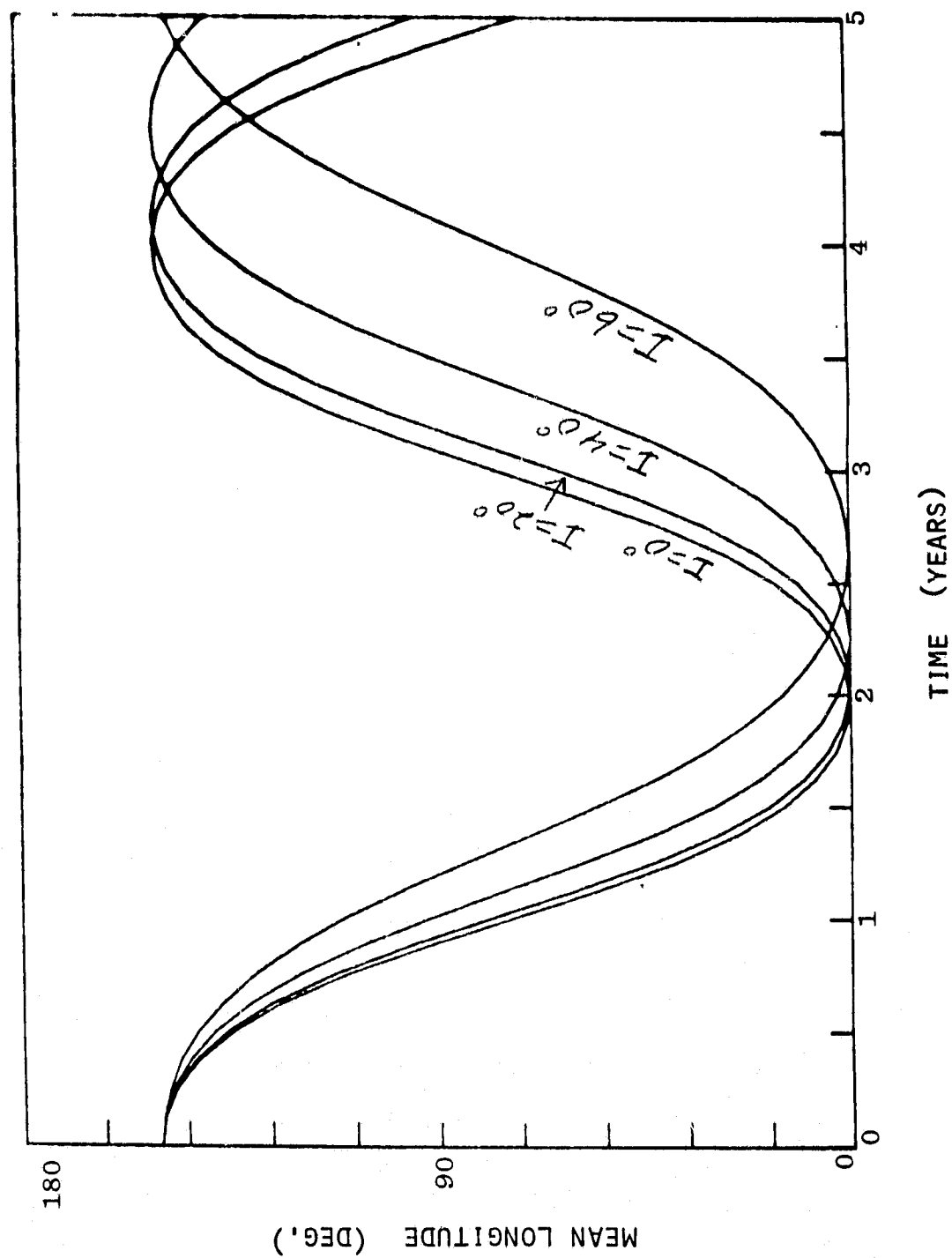


FIGURE 6: MEAN LONGITUDE VS. TIME
(F) INITIAL LONGITUDE = 150°

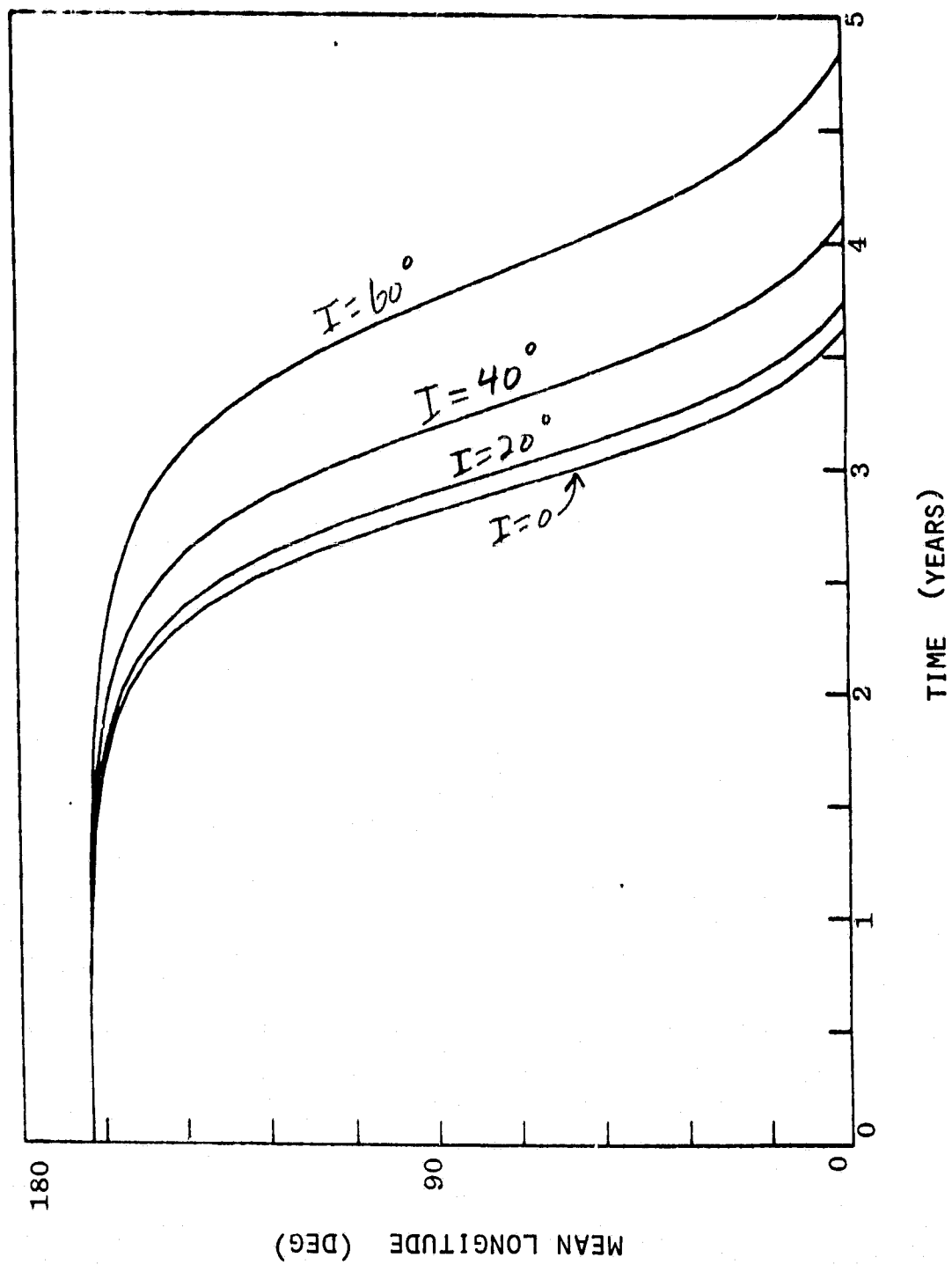


FIGURE 6: MEAN LONGITUDE VS. TIME
(G) INITIAL LONGITUDE = 165°

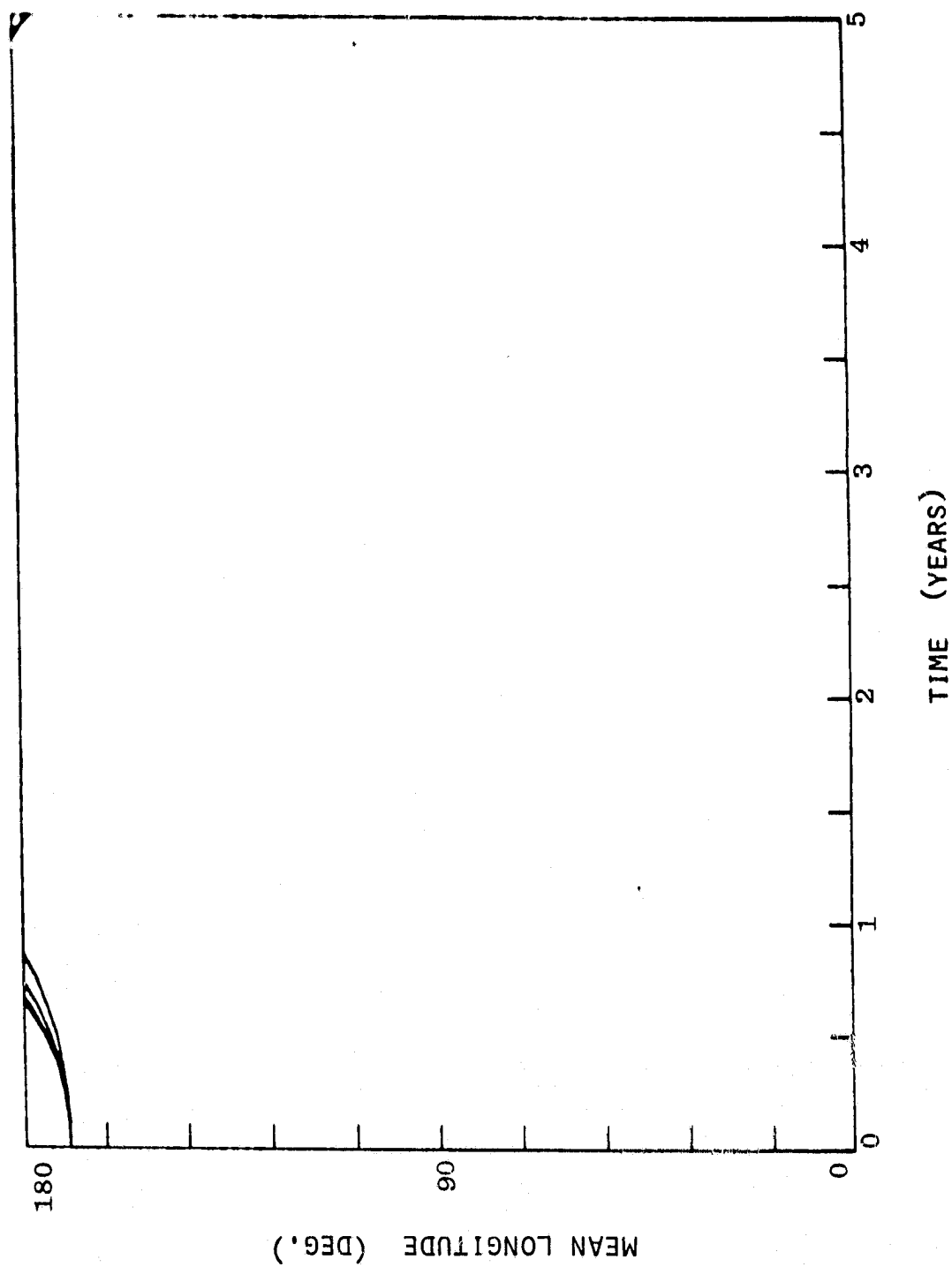


FIGURE 6: MEAN LONGITUDE VS. TIME
(H) INITIAL LONGITUDE = 170°

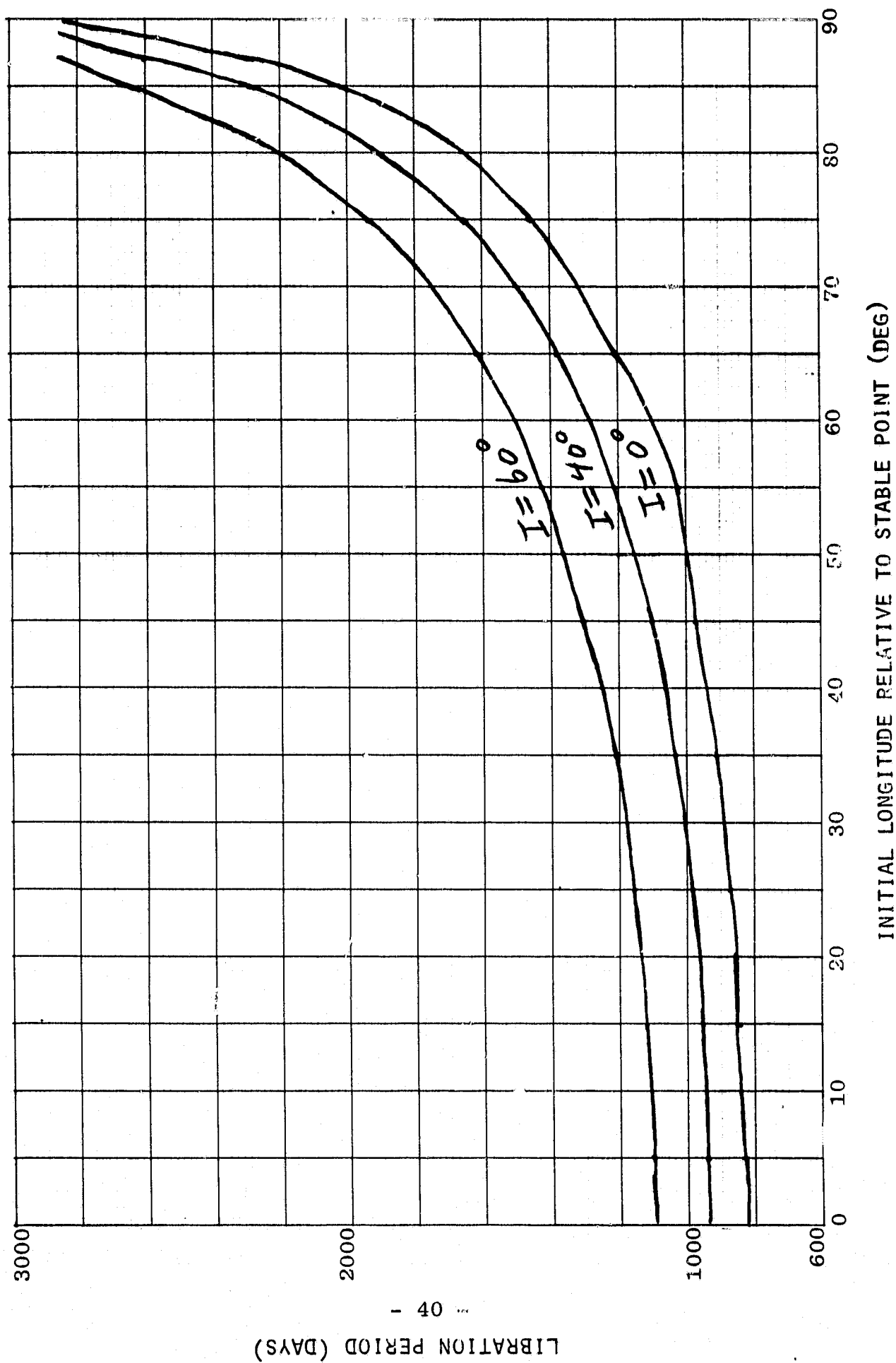


Figure 7: Libration Period for Inclined Orbits

4.1 Motion of the Orbit Plane

The differential equations for the motion of the orbit plane are taken from Reference 11. The equations for inclination I and ascending node Ω are

$$\begin{aligned} \frac{dI}{dE} &= \varepsilon (\beta \cos I \cos \Omega + 2\gamma \sin I \sin 2\Omega), \\ \frac{d\Omega}{dE} &= \frac{\varepsilon}{\sin I} [\beta \cos 2I \cos \Omega - (1 - \gamma \cos 2\Omega) \sin I]. \end{aligned} \quad (4.2)$$

The symbol E is the eccentric anomaly. For small eccentricities, an increase in E of 2π corresponds approximately to one day in time. The parameters in the above equations depend on the gravitational terms of the sun, moon, and J_2 , as well as the obliquity of the ecliptic and the altitude of the satellite. They are non-dimensional parameters with the values

$$\varepsilon = 1.00 \times 10^{-6}, \quad \Omega = .6821, \quad \beta = .1749, \quad \gamma = .0189.$$

The derivation of equations (4.2) and the definition of the parameters are given in Reference 11. Also given there is an analytical solution that is valid for small inclinations.

We are interested here in geosynchronous orbits that may have a large inclination. For this case, there is no analytical solution to (4.2). However, we can solve them by computing a numerical solution. Note that the following integral of motion exists:

$$\begin{aligned} G_3 &= \alpha \sin^2 I - \beta \sin I \cos I \cos \Omega - \\ &\quad - \gamma \sin^3 I \cos 2\Omega. \end{aligned} \quad (4.3)$$

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An advantage of the analysis done in Reference 11 is that concise formulas result. The formulas do not provide high precision, but do provide enough accuracy to adequately describe the motion. That is our purpose in this report, i.e. to determine the nature of the motion of the orbital plane.

In looking at the effects of the perturbations on the orbital plane, we first look at the rate of change of I and Ω . It is of interest to compare ΔI and $\Delta \Omega$ for different inclinations and nodes.

It can be shown that for $I_0 = 0$, then $\Delta I = .841$ deg/year. Compare this with the data in Figure 8. It can be seen that for non-zero inclinations, ΔI is generally less than for the zero inclination case. In fact, when $\Omega_0 = 180^\circ$, then $\Delta I = 0$. However, this does not imply that the orbit will be stable. By observing Figure 9, it is seen that $\Delta \Omega$ can not be zero. Thus as Ω moves, it will induce a motion in I . These considerations will be important in computing out-of-plane orbit correction delta-V requirements in Section 5.

The solutions of (4.2) were obtained on a desk-top calculator, using an Euler method for the numerical integration. Time histories for inclination are shown in Figures 10 for various initial values of inclination and ascending node.

DISCUSSION OF FIGURE 10

- (1) During the course of 10 years, the inclination can change by up to 8 degrees, either increasing or decreasing. The amount of the change depends strongly on the value of the initial ascending node.

- (2) Only one curve is shown for the $I_0 = 0$ case (Figure 10(A)). This is because the ascending node is not defined when the inclination is zero. In Reference 11, it is shown that as the inclination increases, the node becomes defined at 90 deg east of the vernal equinox direction and moves west (i.e. Ω decreases from 90 deg).
- (3) The "inclination offset" method for maintaining inclination is shown in Figure 10(A). Notice that when $I_0 = 1^\circ$ and $\Omega_0 = 270^\circ$, the inclination remains less than 1 deg. for a longer period of time than when $I_0 = 0$.
- (4) The general nature of the inclination time histories are similar for initial inclinations of 40 and 60 degrees.
- (5) Remember from Section 2 that each satellite orbit in a "figure 8 constellation" has a different ascending node. Thus, we see from Figures 10(B) and (C) that the orbital planes of the satellites in the constellation will have a complicated motion relative to one another.

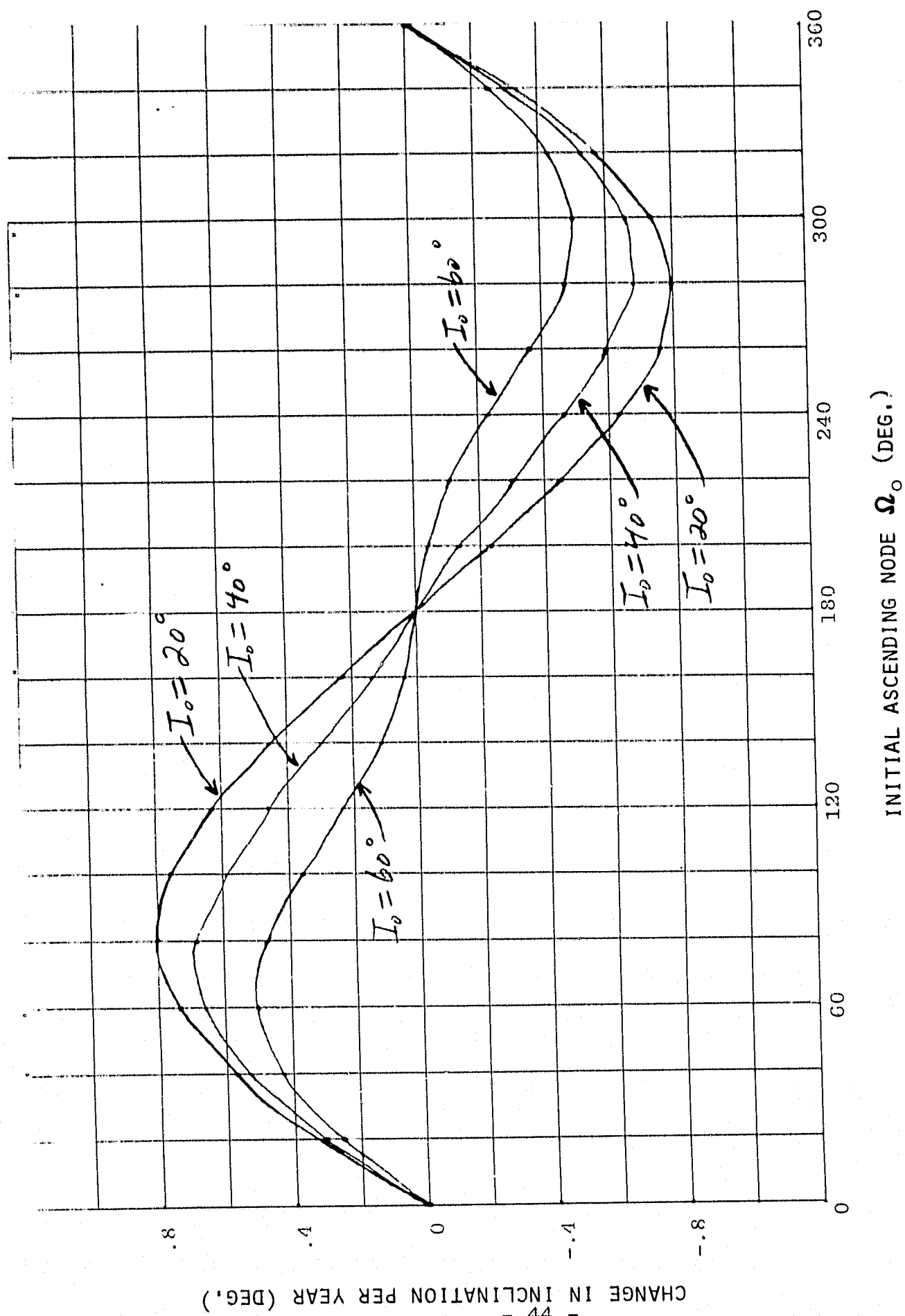


FIGURE 8: RATE OF CHANGE OF INCLINATION

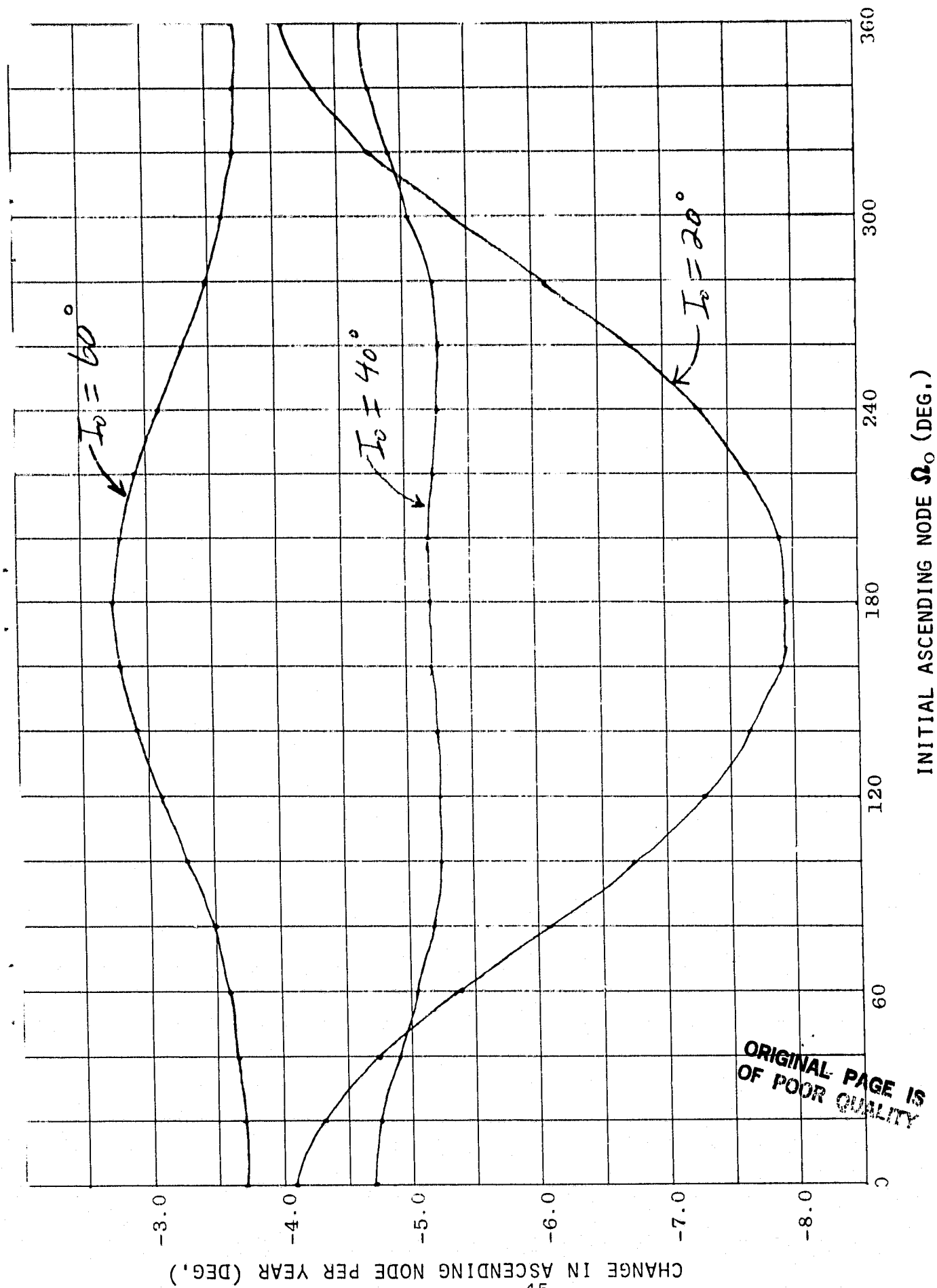


FIGURE 9: RATE OF CHANGE OF ASCENDING NODE

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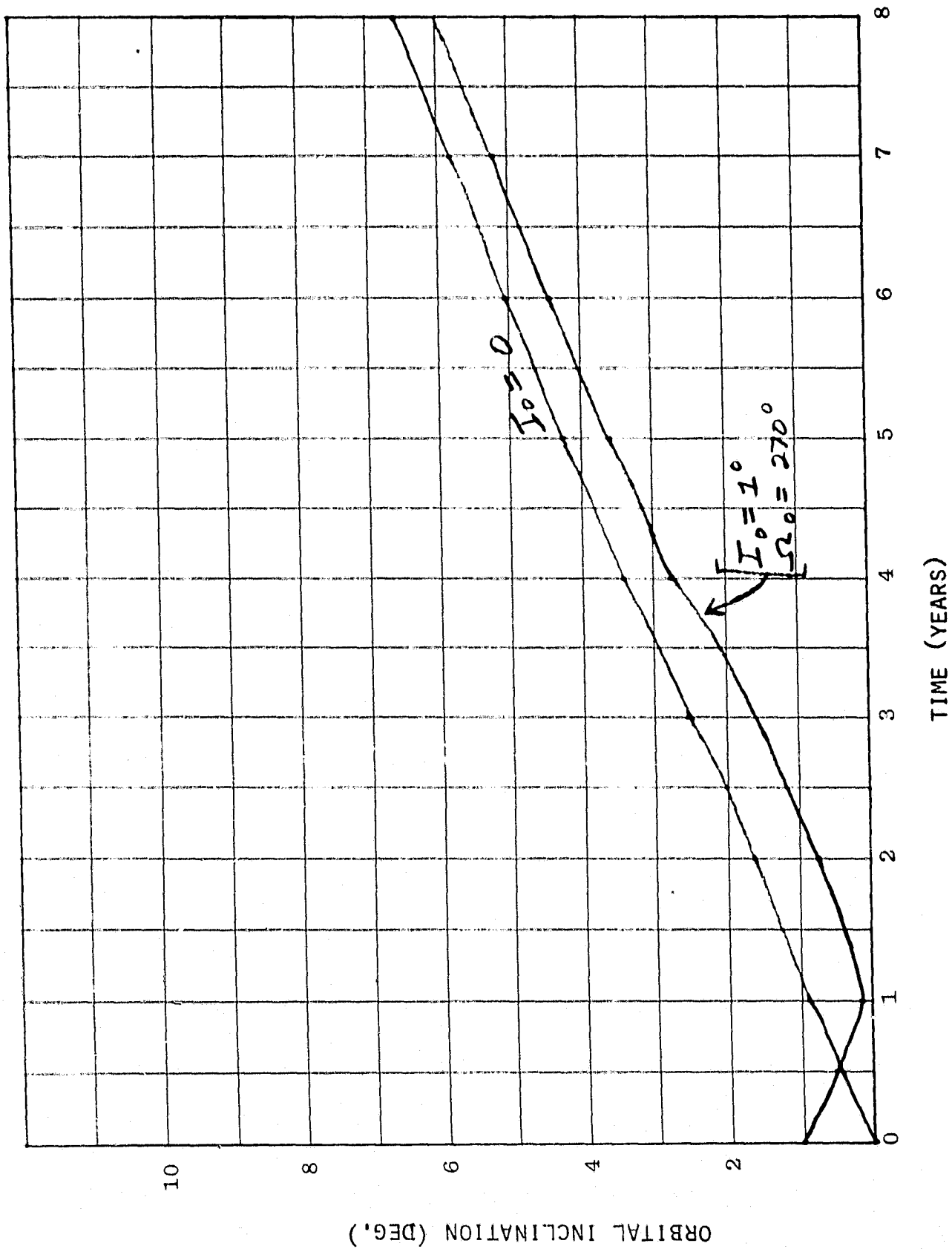


FIGURE 10: INCLINATION TIME HISTORY
(A) $I_0 = 0^\circ$, $I_0 = 1^\circ$

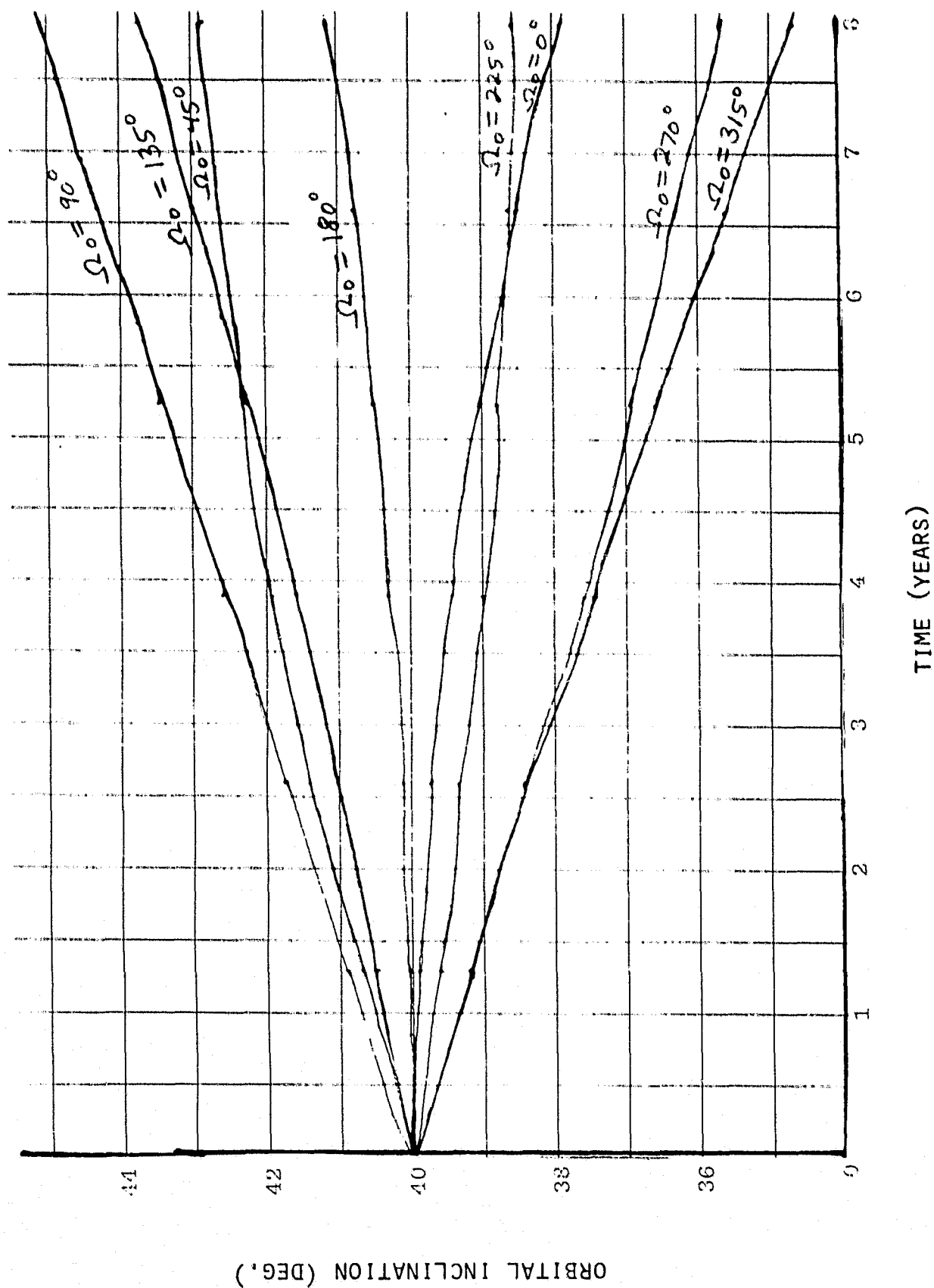


FIGURE 10: INCLINATION TIME HISTORY
(B) $i_0 = 40^\circ$

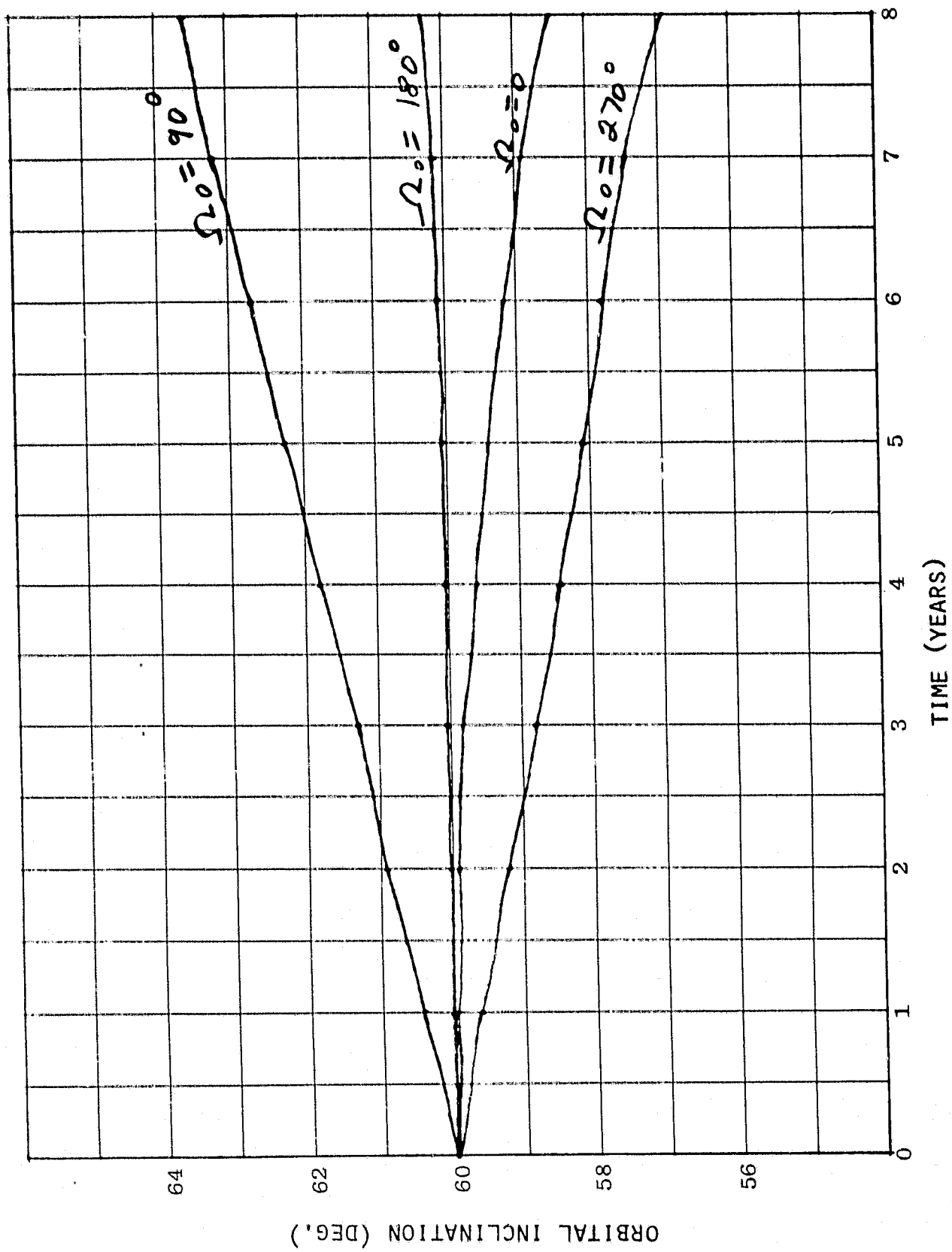


FIGURE 10: INCLINATION TIME HISTORY

(c) $I_o = 60^\circ$

4.3 Change in Eccentricity

As discussed in Section 3.3, the solar radiation pressure will cause the orbital eccentricity to change and the line of apsides (once it can be defined) to rotate. Ordinarily, the solar radiation pressure has a very small effect on the orbit, and is not usually considered in advanced planning studies. However, we will study this perturbation here because solar radiation pressure could become important for geosynchronous satellites with large solar arrays.

The direct gravitational effects of sun, moon and J_2 on eccentricity is small. That is because these terms are proportional to e , which is considered to be small. Therefore, we include only solar radiation pressure effects on orbital eccentricity.

As before, we assume that the eccentricity remains rather small (less than 0.1). Therefore, we can use the equations in Reference 4. Introduce the non-singular elements

$$p = e \cos(u + \Omega), \quad z = e \sin(u + \Omega). \quad (4.4)$$

These elements remain defined for circular orbits.

In developing the acceleration of the satellite due to solar radiation pressure, the following simplifications are made:

- The satellite can be modeled as a flat surface which is perpendicular to the satellite-sun line.
- The satellite is a 10% reflecting body.
- The earth moves on a circular orbit about the sun.
- The solar energy flux is constant in the vicinity of the earth.

Further details on the above described model are given in References 4, 12 and 13.

After making the assumptions and simplification described above, the differential equations for p and q are found in Reference 4 to be

$$\begin{aligned} \frac{dp}{dE} = \frac{3}{2} \varepsilon \left\{ [n' p q + q Q (2L + Qp)] \cos v - \right. \\ \left. - [n^2 G (1 - L^2) + n^2 S p (2 - (L^2 + Q^2))^{1/2} - G q L (2L + Qp) + \right. \\ \left. + S q (2L + Qp) (1 - (Q^2 + L^2)) (2 - (L^2 + Q^2))^{-1/2}] \sin v \right\}, \end{aligned} \quad (4.5)$$

$$\begin{aligned} \frac{dq}{dE} = \frac{3}{2} \varepsilon \left\{ [n^2 (1 - Q^2) - p Q (2L + Qp)] \cos v + \right. \\ \left. - [n^2 G L Q - n^2 S Q (2 - (L^2 + Q^2))^{1/2} + G p L (2L + Qp) - \right. \\ \left. - S p (2L + Qp) (1 - (Q^2 + L^2)) (2 - (L^2 + Q^2))^{-1/2}] \sin v \right\}. \end{aligned}$$

The above equations make use of the following abbreviations:

$$\begin{aligned} L &= \sqrt{1 - \cos I} \cos \Omega, & Q &= -\sqrt{1 - \cos I} \sin \Omega, \\ n^2 &= 1 - (p^2 + q^2), \\ v &= \frac{E}{365.25} + \theta_0. \end{aligned}$$

Also, define the following parameters:

θ_0 = angle of the sun in the ecliptic plane, measured from the vernal equinox.

$$S = \sin(23.5^\circ) ,$$

$$C = \cos(23.5^\circ) ,$$

$$\xi = (5.06 \times 10^{-7}) \frac{A}{W} \left(\frac{r_s}{R_e} \right)^2 . \quad (4.6)$$

A is the cross-sectional area of the satellite in square meters, and W is the weight of the satellite in kilograms. The parameters r_s and R_e have been previously defined.

In order to determine the long term change in eccentricity, the differential equations (4.5) were solved by using a Runge-Kutta numerical integration method. Element time histories are shown in Figures 11, 12 and 13. The data presented in these figures was generated from the EDRIFT program that is listed in Appendix B. This program includes the following routines:

EDRIFT: Main program and numerical integration driver. It also prints the data to the CRT screen and writes the data to an output file. This file is later input to the plot program PLOTIT.

EINPUT: Routine for input and initialization of constants and common blocks.

EDERIV: Routine to compute the right sides of the differential equations (4.5).

RKF45: Runge-Kutta numerical integration routine.

The data in Figures 11, 12 and 13 was generated based on an area to weight ratio of $1.73 \text{ m}^2 (\text{kg})^{-1}$. This is an assumed value for the Solar Power Satellite (Reference 4). However, since the perturbins parameter ξ depends directly on A/W (see equation (4.6)), we can expect that

satellites with different A/W will have proportionate behavior as shown in Figures 11, 12 and 13.

DISCUSSION OF FIGURE 11

- (1) The purpose of these eight figures is to show the eccentricity time history, and how it depends on I and Ω .
- (2) When the orbits lie near the ecliptic plane ($\Omega = 0$), the inclination does not have much effect on eccentricity time history.
- (3) All curves have a period of one year. However, the maximum value of e can depend strongly on inclination and node.

DISCUSSION OF FIGURE 12

- (1) This figure can be thought of as a plot of ρ and φ in polar coordinates (see equations (4.4)). As time increases, the closed curve will be traced in the direction of the arrows, starting from $\rho=0$, $\varphi=0$. The value of the eccentricity will be the distance from the origin. The angle $u+\Omega$ will be the angle between the radius vector and the $\rho=0$ axis. For example, in Figure 12(A), $u+\Omega$ begins at zero degrees and asymptotically approaches 180 deg., to start the cycle again.
- (2) For different inclinations and nodes, the shape of the closed curve will change. In the $I=0$ case, the curve is slightly flattened. This is because the orbit is inclined to the ecliptic plane (the plane of the sun).
- (3) For a node of 90 deg. and inclinations of 20, 40 and 60 deg., the curve is flattened. Compare with Figure 11(C).

DISCUSSION OF FIGURE 13

The purpose of the figure is to show the long term (10 years) motion of eccentricity. Note that the motion is periodic with a period of one year. Thus, it is only necessary to look at the motion within one year.

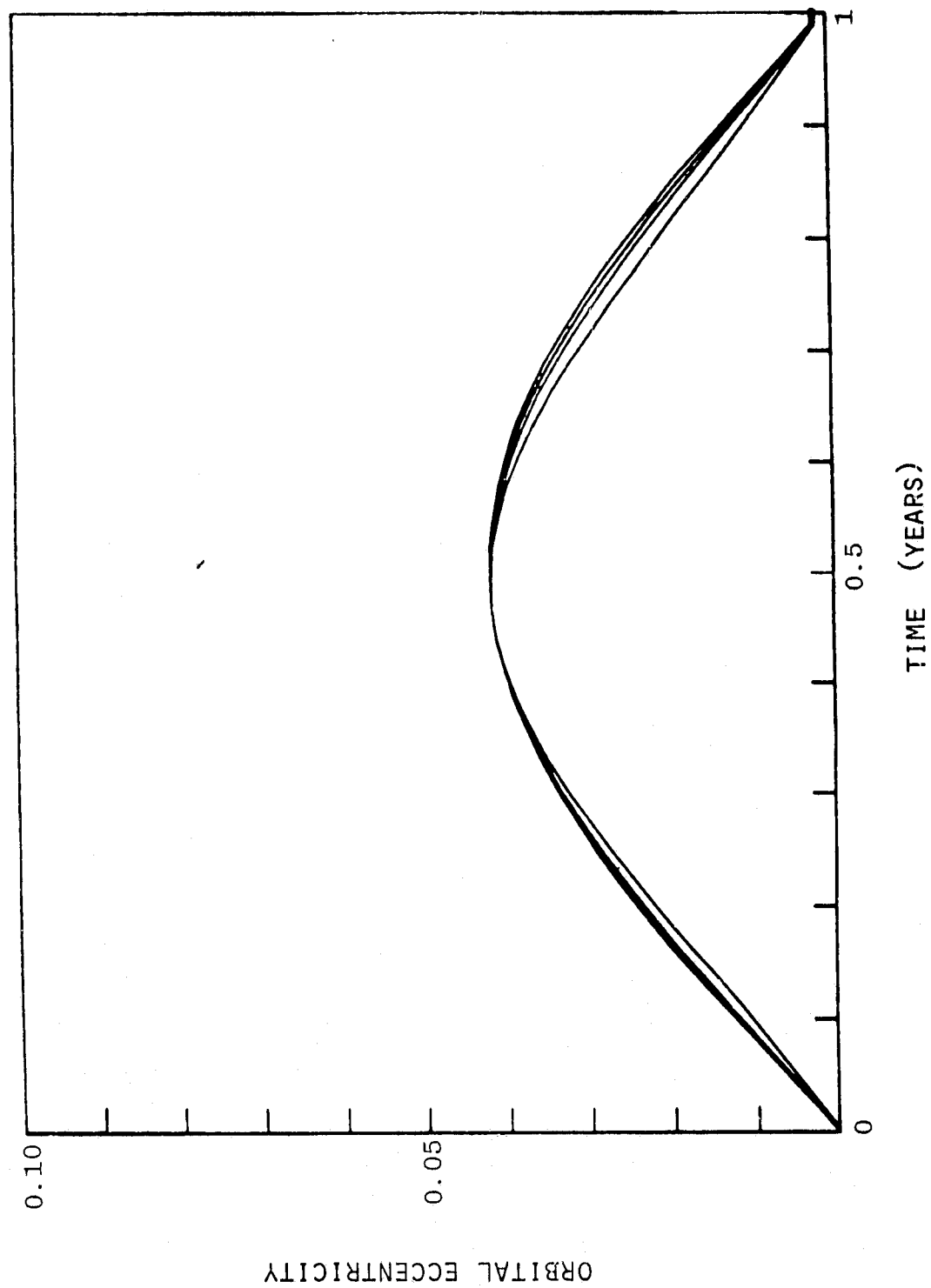


Figure 11: Eccentricity Time History
(a) $\Omega_0 = 0$

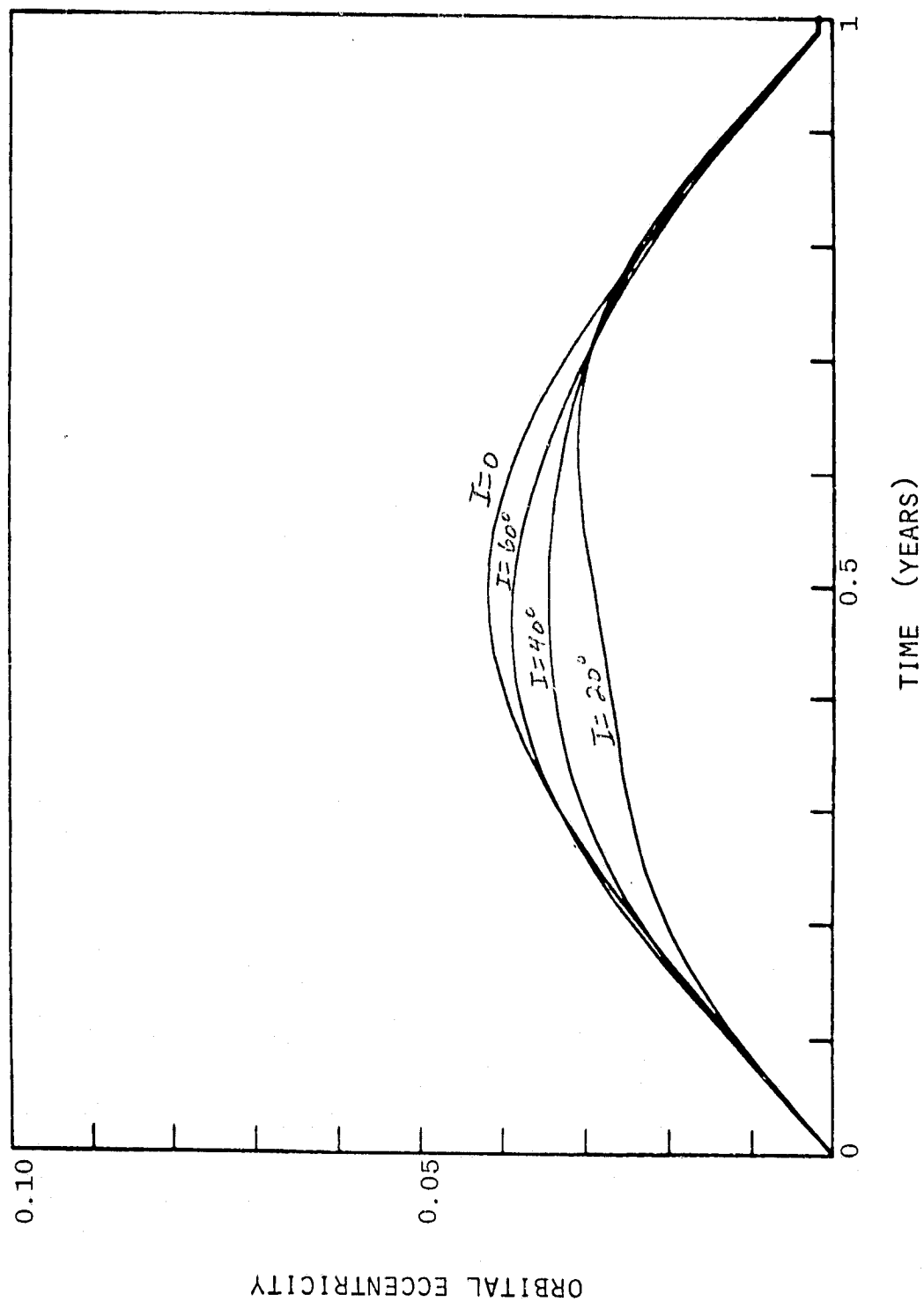


Figure 11: Eccentricity Time History
(b) $\Omega = 45^\circ$

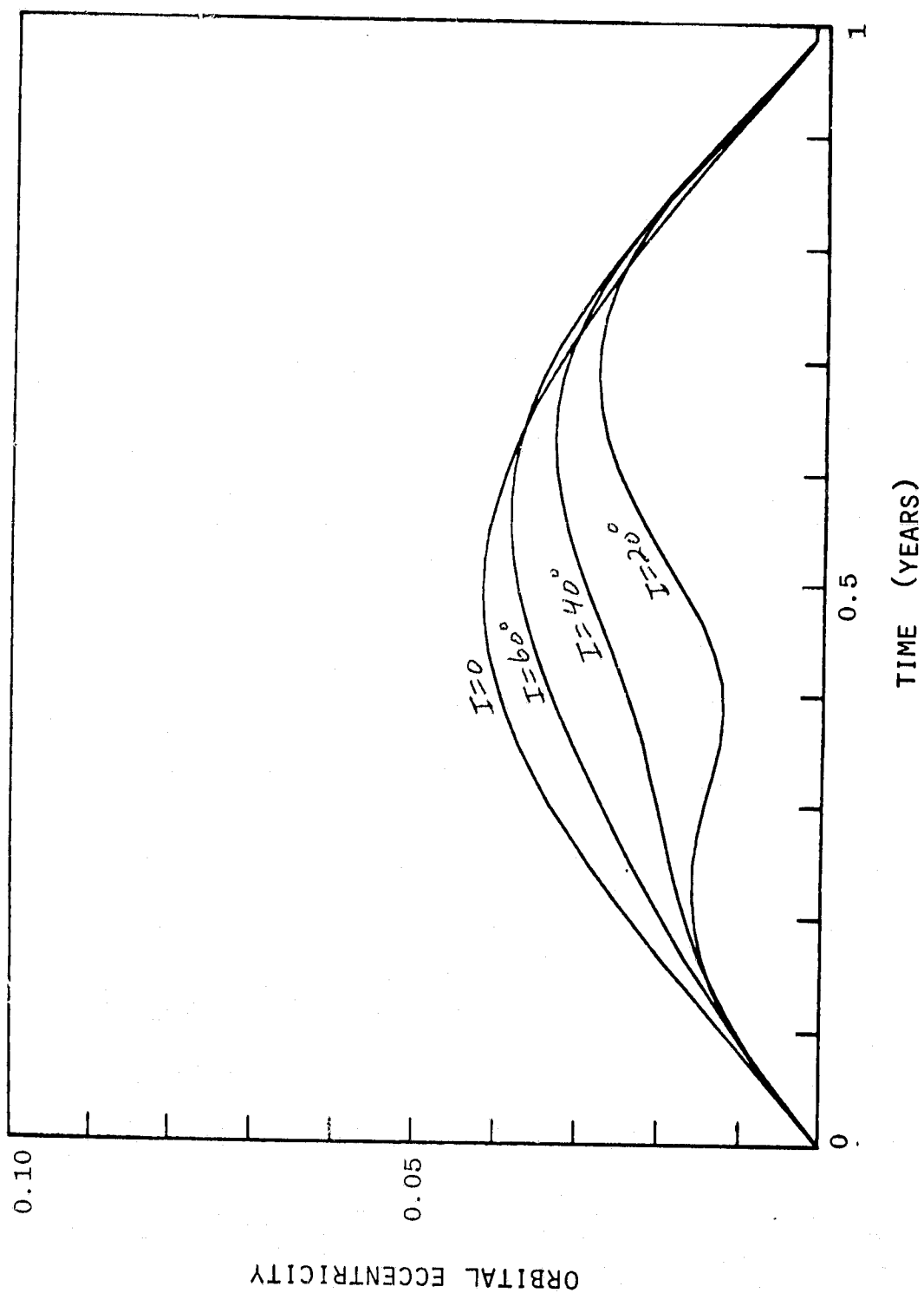


Figure 11: Eccentricity Time History
(c) $\Omega = 90^\circ$

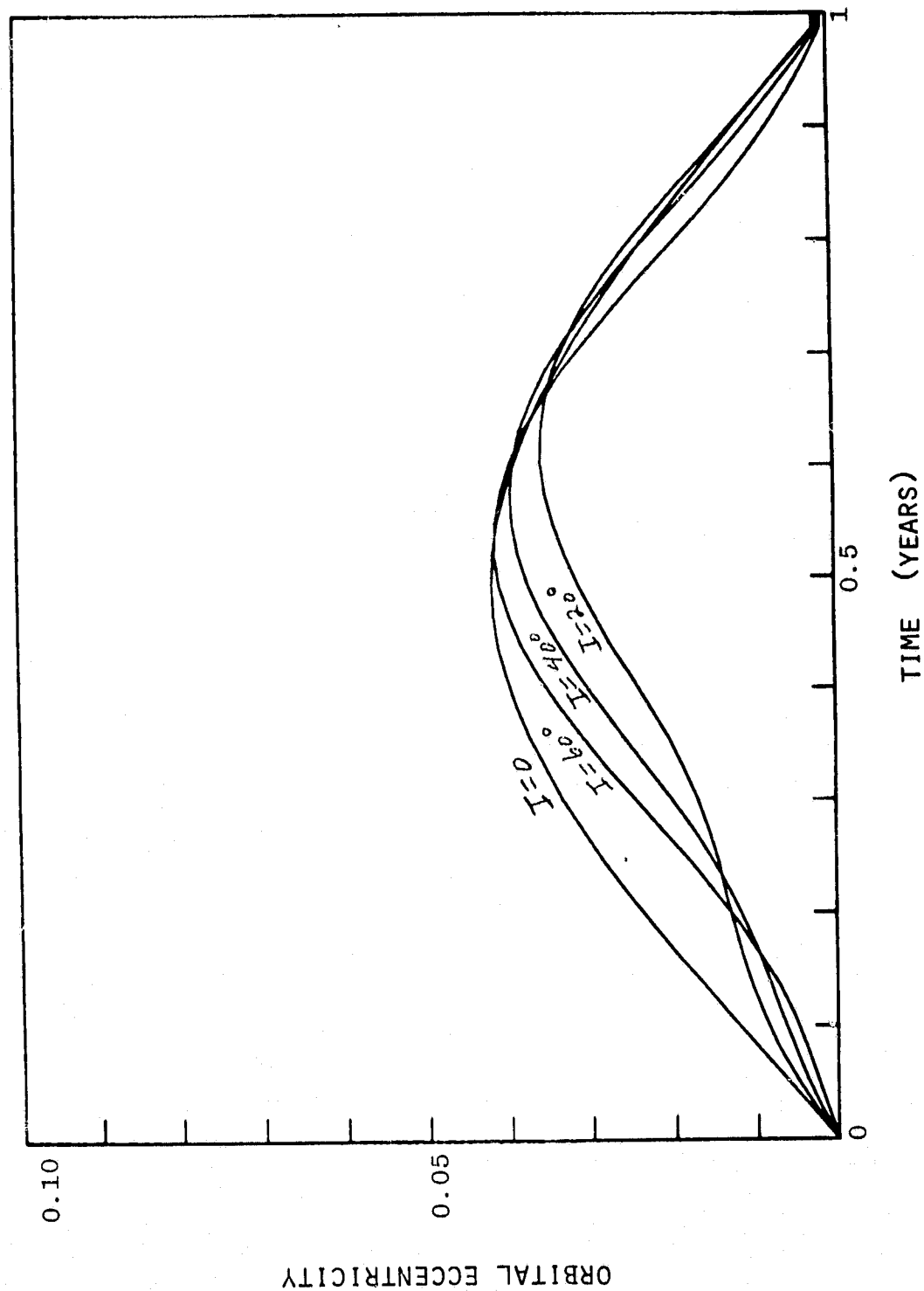


Figure 11: Eccentricity Time History
(d) $\Omega = 135^\circ$

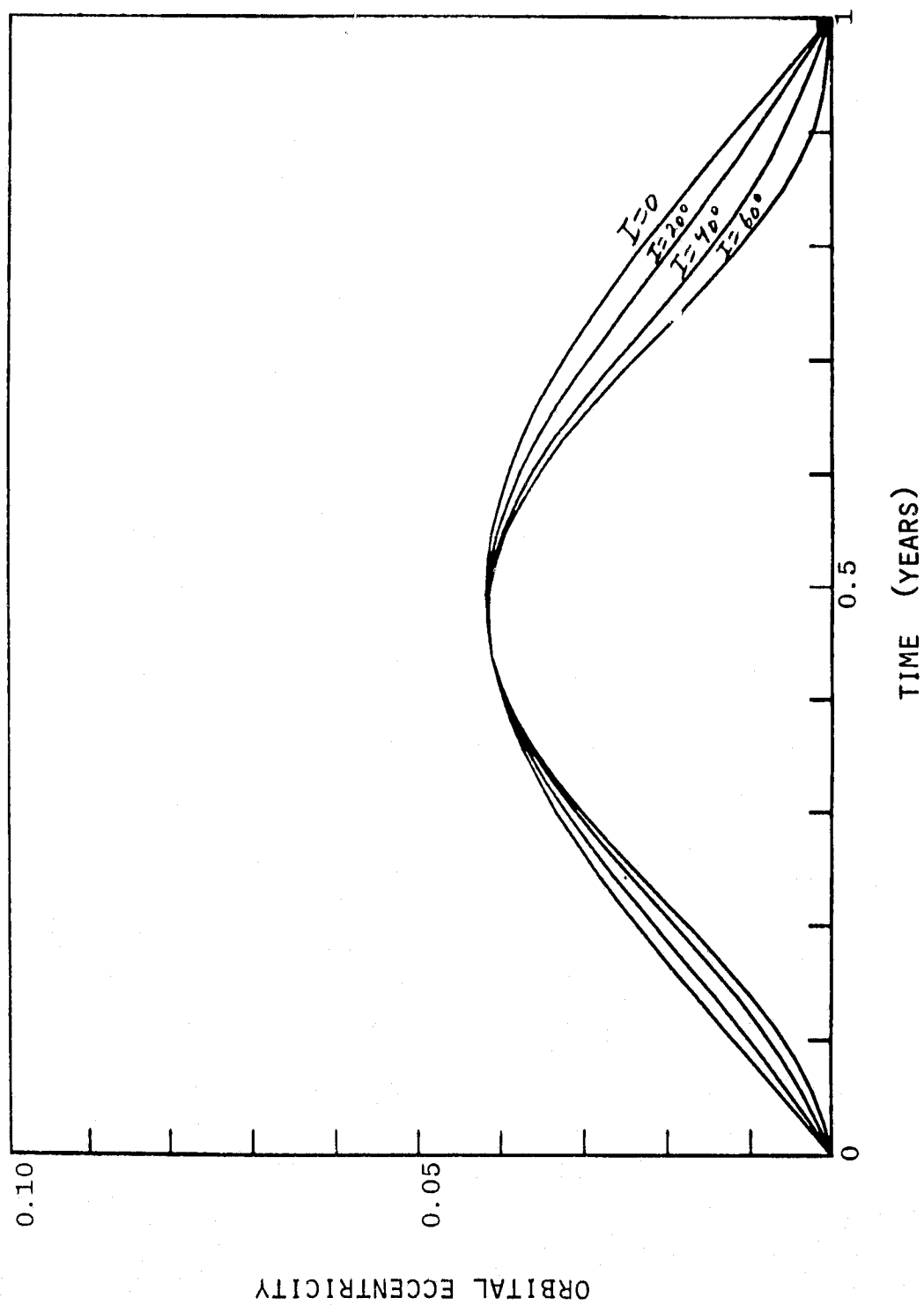


Figure 11: Eccentricity Time History
(e) $\Omega = 180^\circ$

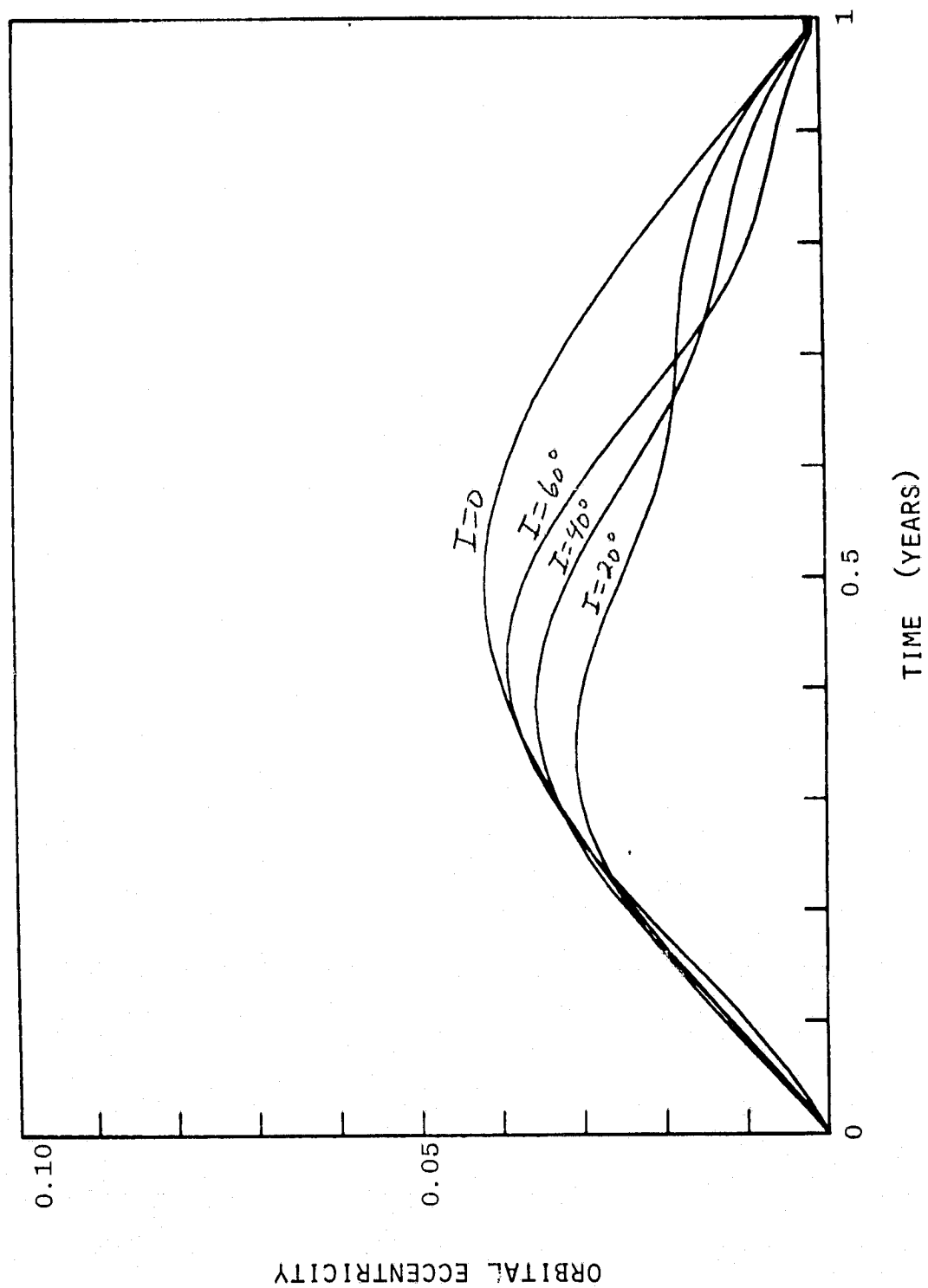


Figure 11: Eccentricity Time History
(f) $\Omega = 225^\circ$

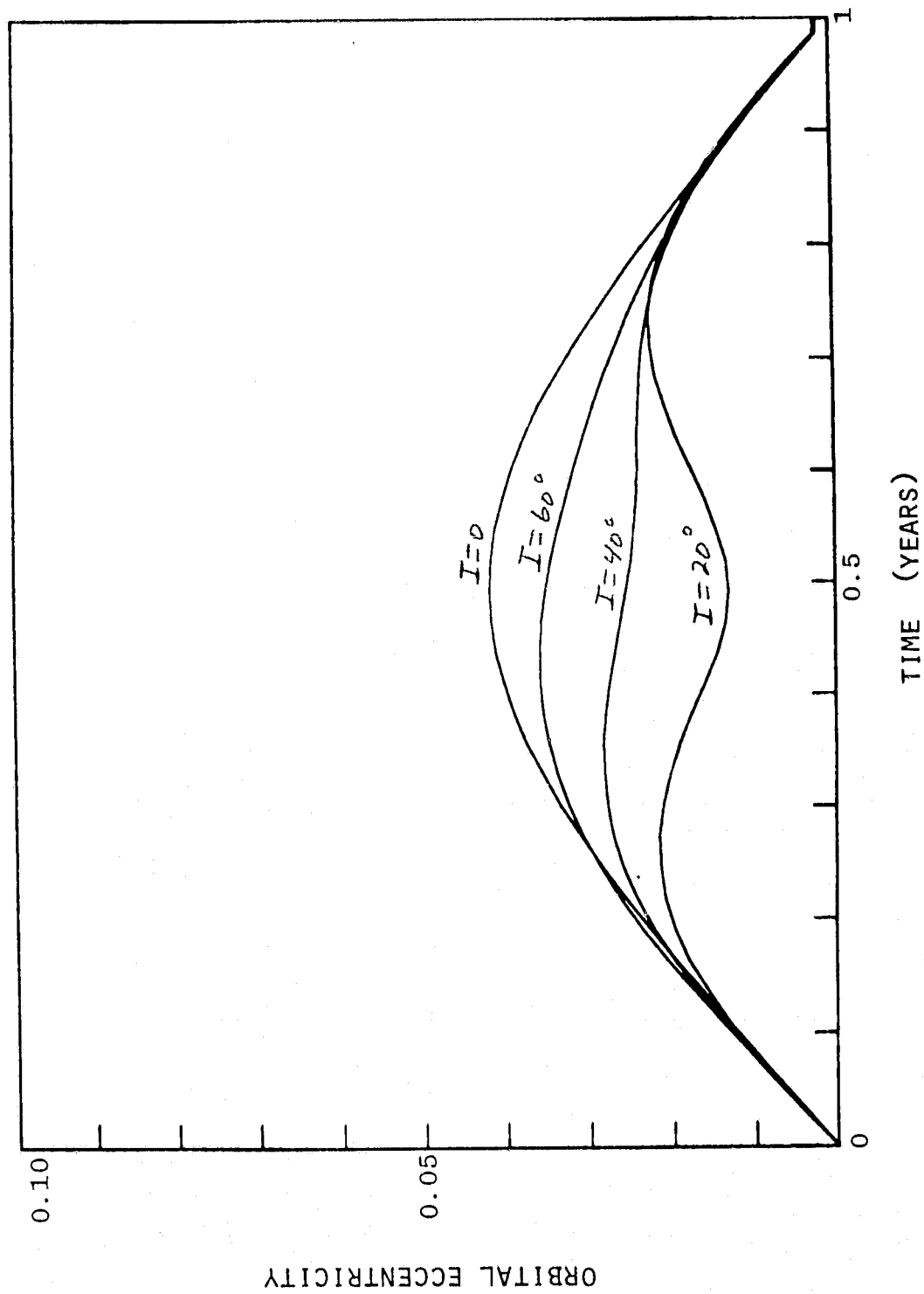


Figure 11: Eccentricity Time History
(g) $\Omega = 270^\circ$

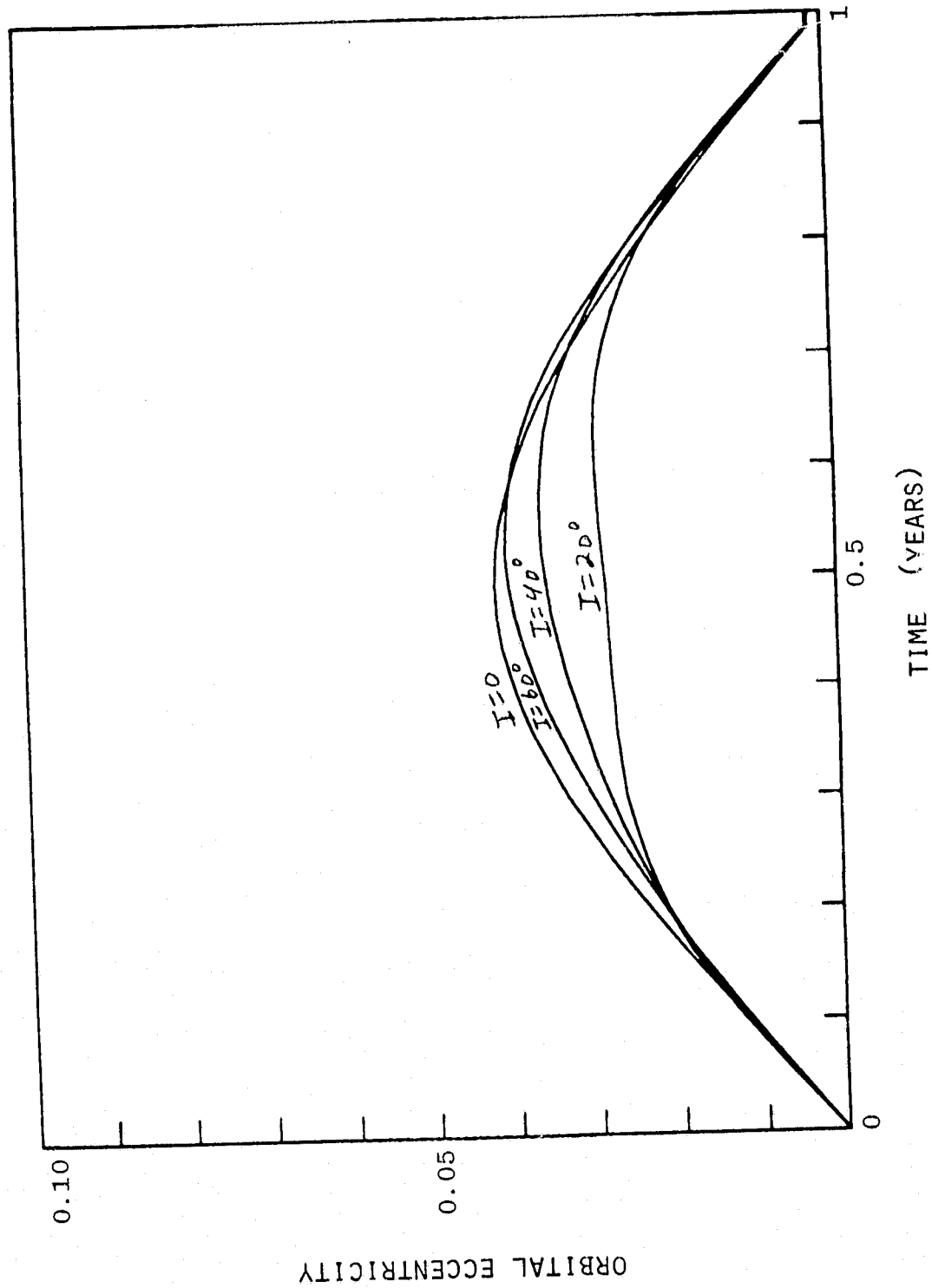


Figure 11: Eccentricity Time History
(h) $\Omega = 315^\circ$

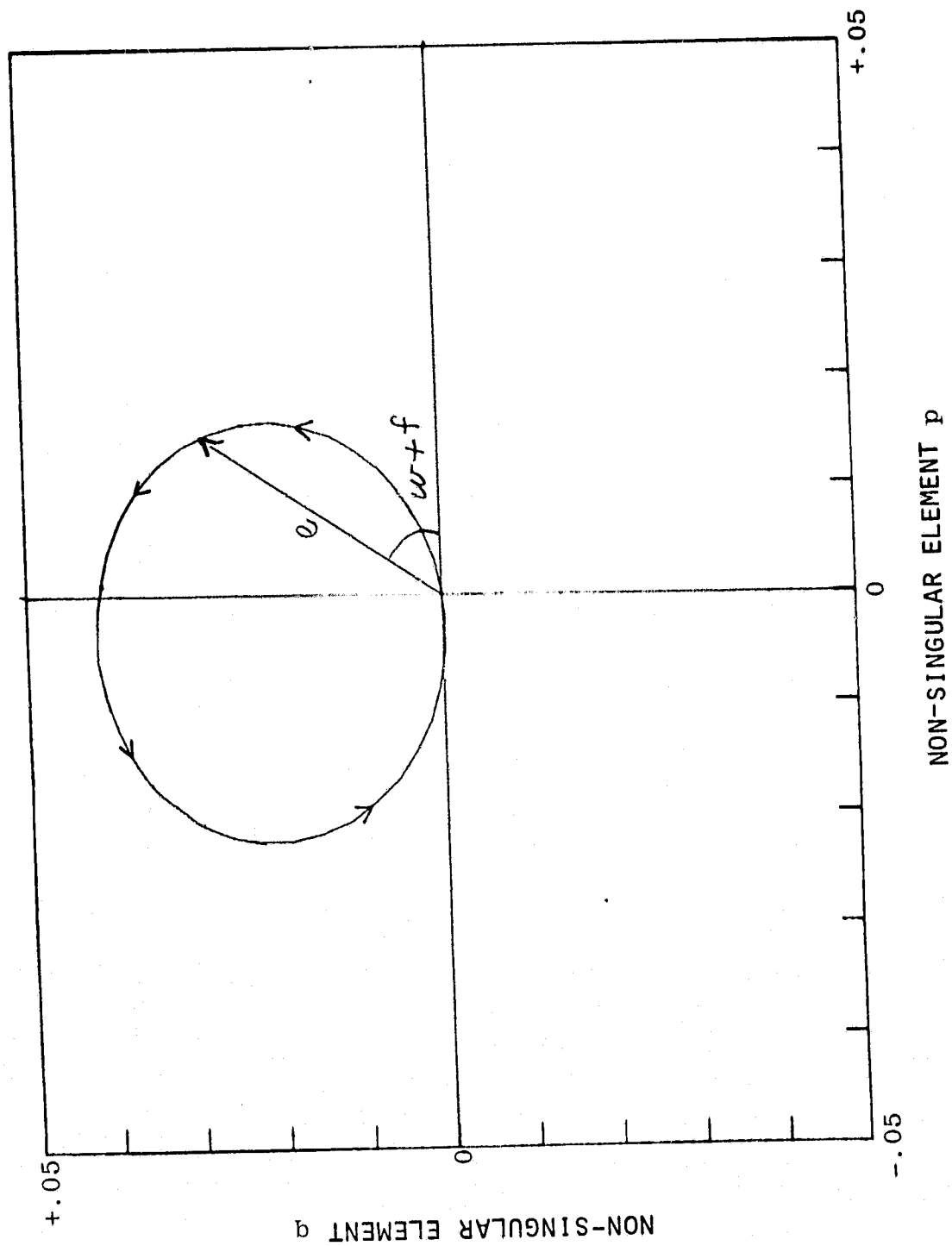


Figure 12: Motion of Eccentricity and Line of Apsides
(a) $I = 0^\circ$

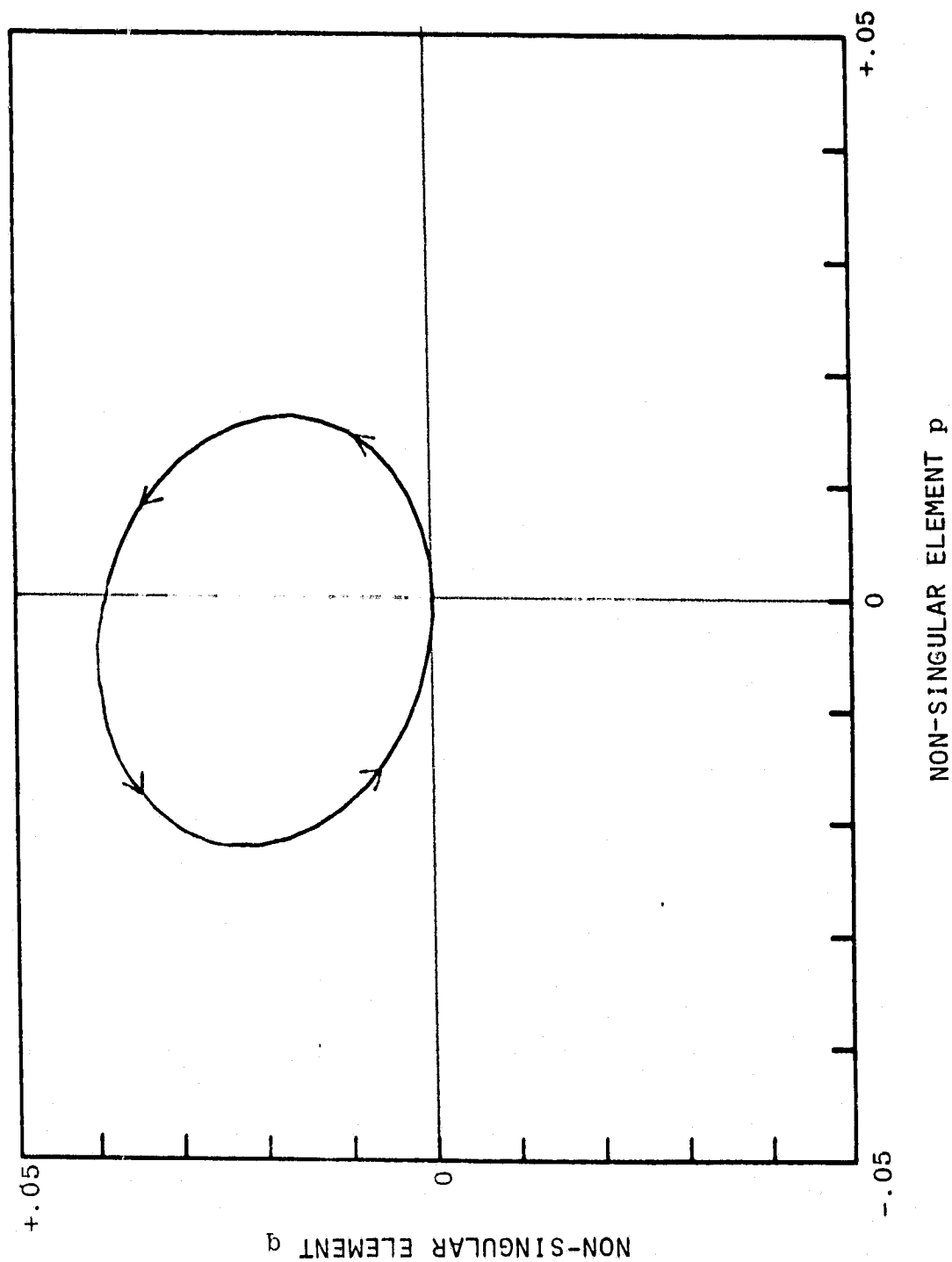


Figure 12: Motion of Eccentricity and Line of Apsides
(b) $I = 20^\circ$, $\Omega = 90^\circ$

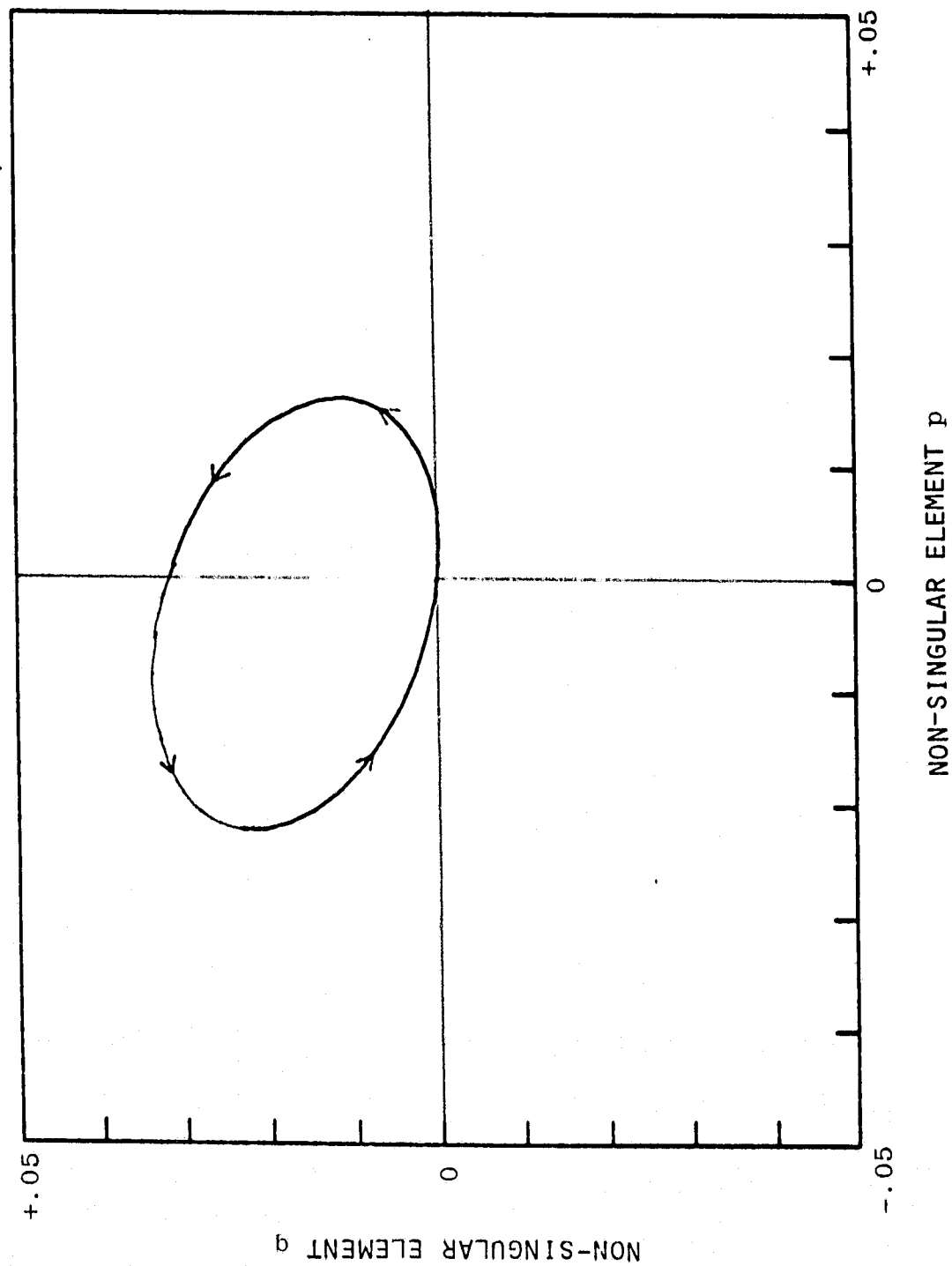


Figure 12: Motion of Eccentricity and Line of Apsides
(c) $I = 40^\circ$, $\Omega = 90^\circ$

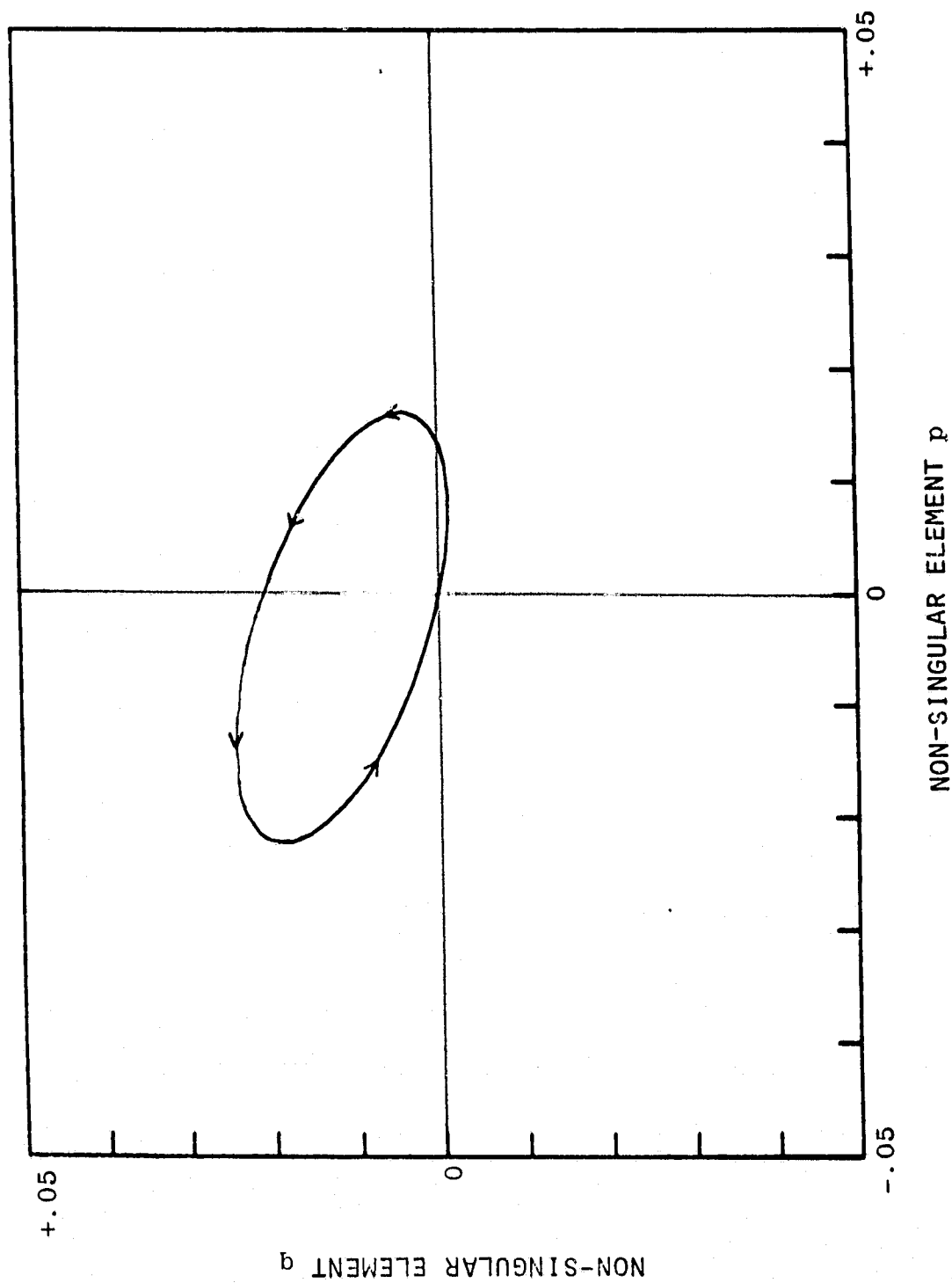


Figure 12: Motion of Eccentricity and Line of Apsides
(d) $I = 60^\circ$, $\Omega = 90^\circ$

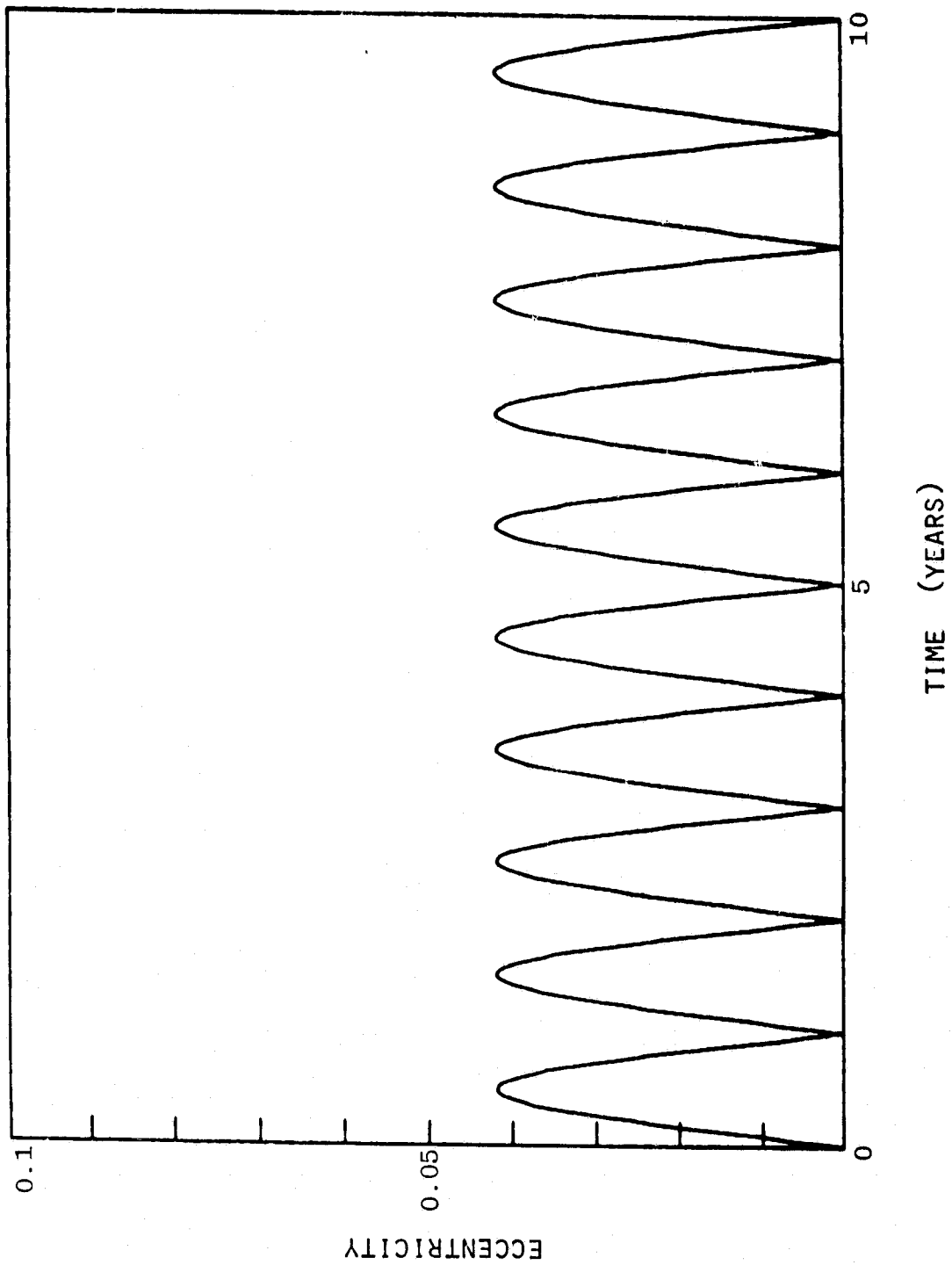


Figure 13: Long Term Motion of Eccentricity

5.0 VELOCITY REQUIREMENTS FOR ORBIT MAINTENANCE

In this section we determine the delta velocity increments that are required to offset the orbit perturbations.

5.1 In-Plane Delta-V

5.1.1 Mathematical Developments

This analysis presented in this section makes use of equations and results from References 10, and 14. The following two assumptions are made:

- (1) The orbit is nearly circular. Thus the delta-V (ΔV) in the radial direction is negligible as compared to the tangential component.
- (2) There is no coupling between the in-plane and out-of-plane perturbations. This is a limitation of the analytical perturbation method, and was discussed in the previous section.

The orbit-averaged longitude drift of an inclined circular orbit was given in Section 4.1 as

$$\ddot{\lambda} = A_{22} \sin 2(\lambda - \lambda_{22}). \quad (5.1)$$

where

$$A_{22} = -\frac{3}{r_3} F_{22} F(I)_{22} \left(\frac{\text{rad}}{\text{sid. day}} \right)^2.$$

Since we are interested in the deviation of the mean longitude of ascending node of geosynchronous orbits, it is convenient to linearize the above equation in terms of the variation $\Delta \lambda$ defined by the relation

$$\lambda = \lambda_s + \Delta\lambda \quad (5.2)$$

where λ_s is the desired mean longitude of ascending node of the orbit.

By substitution of (5.2) into (5.1) we set

$$\Delta\ddot{\lambda} + [-2A_{22} \cos 2(\lambda_s - \lambda_{22})] \Delta\lambda = A_{22} \sin 2(\lambda_s - \lambda_{22}). \quad (5.3)$$

The solution to the corresponding homogeneous equation of (5.3) is

$$\Delta\lambda_H = C_1 \cos \omega t + C_2 \sin \omega t,$$

where

$$\omega = [-2A_{22} \cos 2(\lambda_s - \lambda_{22})]^{1/2}.$$

The particular integral can be found by assuming

$$\Delta\lambda_p = C_3$$

Substituting into equation (5.3) and solving for C_3 we set

$$C_3 = -\frac{1}{2} \tan 2(\lambda_s - \lambda_{22}).$$

Therefore, the complete solution is

$$\Delta\lambda = C_1 \cos \omega t + C_2 \sin \omega t - \frac{1}{2} \tan 2(\lambda_s - \lambda_{22}).$$

The integration constants can be found by assuming there is no initial orbit injection error, so that $\lambda = 0$

and $\Delta\dot{\lambda} = 0$ at $t = 0$:

(1) For $t = 0$ and $\Delta\lambda = 0$, then

$$C_1 = \frac{1}{2} \tan 2(\lambda_s - \lambda_{22}).$$

(2) For $t = 0$ and $\Delta\dot{\lambda} = 0$, then

$$C_2 = 0$$

Therefore the solution becomes,

$$\Delta\lambda = \frac{1}{2} \tan 2(\lambda_s - \lambda_{22}) [\cos \omega t - 1]. \quad (5.4)$$

Since w is small, equation (5.4) can be further simplified by expanding $\cos w\tau$ and inserting the expression for w . The result is

$$\Delta\lambda = -A_{22}\tau^2 \sin 2(\lambda_s - \lambda_{22}) \quad (5.5)$$

In Reference 10 it is shown that the rate of change of the circular synchronous orbit radius satisfies the equation

$$\dot{r} = B_{22} \sin 2(\lambda - \lambda_{22}), \quad (5.6)$$

where

$$B_{22} = 24 \pi r_s \left(\frac{R_c}{r_s} \right)^2 J_{22} F(I)_{22}.$$

Equation (5.6) can be linearized in terms of variations $\Delta\lambda$ and Δr , which is defined by the relation

$$r = r_s + \Delta r.$$

Substitute the expressions involving Δr and $\Delta\lambda$ into equation (5.6) to get

$$\Delta \dot{r} = B_{22} \sin 2(\lambda_s - \lambda_{22}) + 2B_{22} \cos 2(\lambda_s - \lambda_{22}) \Delta\lambda. \quad (5.7)$$

By substituting equation (5.5) into (5.7)

$$\begin{aligned} \Delta \dot{r} = & B_{22} \sin 2(\lambda_s - \lambda_{22}) + \\ & + 2B_{22} \cos 2(\lambda_s - \lambda_{22}) [-A_{22}\tau^2 \sin 2(\lambda_s - \lambda_{22})]. \end{aligned} \quad (5.8)$$

Solving the differential equation (5.8)

$$\begin{aligned} \Delta r = & B_{22} \sin 2(\lambda_s - \lambda_{22}) \tau - \\ & - \frac{2}{3} B_{22} \cos 2(\lambda_s - \lambda_{22}) A_{22} \tau^3 \sin 2(\lambda_s - \lambda_{22}) + C. \end{aligned} \quad (5.9)$$

As before, assume that there is no initial injection error, i.e. for $\tau=0$ and $\Delta r=0$, then $C=0$.

We thus have the expression:

$$\Delta r = B_{22} \tau \sin 2(\lambda_s - \lambda_{22}) \left[1 - \frac{2}{3} A_{22} \tau^2 \cos 2(\lambda_s - \lambda_{22}) \right]. \quad (5.10)$$

To determine the station-keeping ΔV requirements, consider a satellite which has been drifting, such that $\Delta\lambda = \Delta\lambda_0$. By using equation (5.5), the time of drift can be found. At that time it will have a radial distance of $r_s + \Delta r$. For a near circular orbit the velocity relative to the inertial space is given by

$$V_1 = (r_s + \Delta r) \dot{\phi}_e - r_s \Delta \dot{f} \quad (5.11)$$

where $\Delta \dot{f}$ is the variation in true anomaly along the orbit, and $\dot{\phi}_e$ is the rotational rate of the earth.

To cause the satellite to return to the original value of λ_s , it will be necessary to transfer to a circular orbit of radius $r_s - \Delta r$ with a velocity V_2 relative to the inertial space given by

$$V_2 = (r_s - \Delta r) \dot{\phi}_e + r_s \Delta \dot{f} \quad (5.12)$$

In this new orbit, the satellite will drift back to λ_s , and then begin the cycle again. (Note that this takes advantage of the linearity for small deviations.)

The minimum energy coplanar circular orbits transfer is a Hohmann transfer ellipse with apogee and perigee distances of $r_s + \Delta r$ and $r_s - \Delta r$, respectively. The two velocity impulses are

$$\Delta V_1 = V_1 \left(\sqrt{\frac{2}{1 + \frac{r_s + \Delta r}{r_s - \Delta r}}} - 1 \right) \quad (5.13)$$

$$\Delta V_1 \approx -\frac{1}{2} V_1 \frac{\Delta r}{r_s}.$$

$$\Delta V_2 = V_2 \left(1 - \sqrt{\frac{2 \left(\frac{r_3 + \Delta r}{r_3 - \Delta r} \right)}{1 + \left(\frac{r_3 + \Delta r}{r_3 - \Delta r} \right)}} \right), \quad (5.14)$$

$$\Delta V_2 \cong -\frac{1}{2} V_2 \frac{\Delta r}{r_3}.$$

Note that these two impulses are applied at one half a sidereal day apart.

By substituting equations (5.11) and (5.12) into (5.13) and (5.14), we get

$$\Delta V_1 = -\frac{1}{2} \left[\Delta r \dot{\phi}_e + \frac{1}{r_3} (\Delta r)^2 \dot{\phi}_e - \Delta \dot{f} \Delta r \right],$$

$$\Delta V_2 = \frac{1}{2} \left[\Delta r \dot{\phi}_e - \frac{1}{r_3} (\Delta r)^2 \dot{\phi}_e + \Delta \dot{f} \Delta r \right].$$

By neglecting second and higher order terms of the small variations, we set

$$\Delta V_1 \cong -\frac{1}{2} \Delta r \dot{\phi}_e,$$

$$\Delta V_2 \cong \frac{1}{2} \Delta r \dot{\phi}_e.$$

The station-keeping ΔV requirement is then

$$\Delta V = |\Delta V_1| + |\Delta V_2| = |\Delta r \dot{\phi}_e|. \quad (5.15)$$

The time period for the cycle is equal to $2T$, where from (5.5)

$$T = \left[\Delta \lambda_0 / (-A_{22} \sin 2(\lambda_5 - \lambda_{22})) \right]^{1/2}.$$

In equation (5.10) the second term in the right hand side is on the order of 10^{-3} of the first term. If we neglect second term in equation (5.10), we can write ΔV as

$$\frac{\Delta V}{2T} = \left| \frac{1}{2} B_{22} \sin 2(\lambda_5 - \lambda_{22}) \dot{\phi}_e \right|, \quad (5.16)$$

which is independent of the time period of the correction cycle.

5.1.2 Numerical Results

The equations to compute in-plane delta V have been implemented in program DELTAV (Appendix B). Results from that program are given here.

The in-plane delta V requirements are shown in Figure 14. The longitude is measured relative to the stable points (105 deg W and 75 deg E). There will be four longitudes which require zero delta V (the two stable and two unstable equilibrium positions). The overall effect for inclined orbits is to decrease the delta V requirements. This goes back to the "inclination factor" in the expression for $\ddot{\lambda}$.

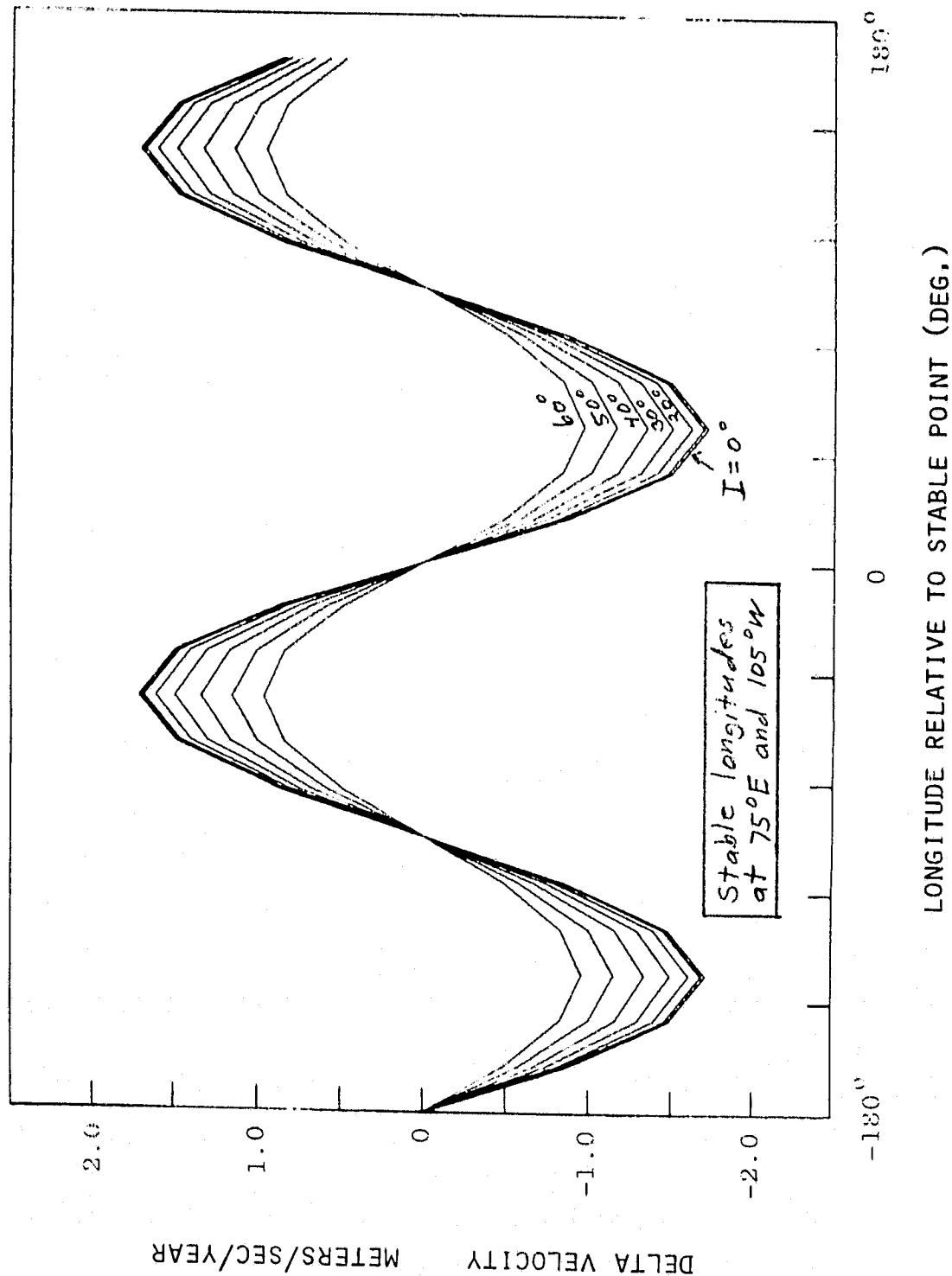


FIGURE 14: IN-PLANE VELOCITY CORRECTION FOR LONGITUDE DRIFT

5.2 Out-of-Plane Delta V

This section will discuss the delta V requirements to correct for the perturbations of the orbital plane. These perturbations were discussed in Section 4.2.

5.2.1 Mathematical Developments

The normal to the orbit plane can be represented in inertial coordinates as

$$\hat{U} = \hat{i} \sin I \sin \Omega - \hat{j} \sin I \cos \Omega + \hat{k} \cos I$$

where \hat{i} , \hat{j} and \hat{k} are unit vectors in the x, y, z axes, respectively.

If the orbital plane has been perturbed through the small angles $\Delta \Omega$ and ΔI , then the normal to this perturbed plane can be represented in inertial coordinates as

$$\hat{U}' = \hat{i} \sin(I + \Delta I) \sin(\Omega + \Delta \Omega) - \hat{j} \sin(I + \Delta I) \cos(\Omega + \Delta \Omega) + \hat{k} \cos(I + \Delta I).$$

Expanding and neglecting higher order terms in ΔI and $\Delta \Omega$:

$$\begin{aligned} \hat{U}' = & \hat{i} (\sin I \sin \Omega + \Delta \Omega \sin I \cos \Omega + \Delta I \cos I \sin \Omega) + \\ & + \hat{j} (-\sin I \cos \Omega + \Delta \Omega \sin I \sin \Omega - \Delta I \cos I \cos \Omega) + \\ & + \hat{k} (\cos I - \Delta I \sin I). \end{aligned}$$

The angular momentum of a geosynchronous satellite in the original orbital plane is

$$\vec{h} = r^2 \dot{\phi} \hat{U}.$$

The angular momentum of a geosynchronous satellite in the perturbed plane is

$$\vec{h}' = r^2 \dot{\phi} \hat{U}'.$$

Therefore the change in the angular momentum vector is

$$\begin{aligned} \Delta \vec{h} = & r^2 \dot{\phi} \left[\hat{i} (\Delta \Omega \sin I \cos \Omega + \Delta I \cos I \sin \Omega) + \right. \\ & + \hat{j} (\Delta \Omega \sin I \sin \Omega - \Delta I \cos I \cos \Omega) - \\ & \left. - \hat{k} (\Delta I \sin I) \right]. \end{aligned}$$

The magnitude of $\Delta \vec{h}$ can be approximated by

$$|\Delta \vec{h}| = r^2 \dot{f} [(\Delta I - \Delta \Omega \sin I)^2 + 2 \Delta I \Delta \Omega \sin I]^{1/2}$$

The relation between Δh and ΔV is given by

$$\Delta h = r \Delta V$$

If we make the following assumptions

$$\Delta I = \dot{I} \Delta T, \quad \Delta \Omega = \dot{\Omega} \Delta T,$$

then the delta V requirement per unit time is given by

$$\frac{\Delta V}{\Delta T} = r \dot{f} [(\dot{I} - \dot{\Omega} \sin I)^2 + 2 \dot{I} \dot{\Omega} \sin I]^{1/2}$$

where \dot{I} and $\dot{\Omega}$ are given by equation (4.2).

5.2.2 Numerical Results

The equations to compute out-of-plane delta V requirements have been implemented in program DELTAVN (Appendix B). Results from that program are given here.

The out-of-plane delta V requirements are presented graphically in Figures 15(A) and 15(B). In the first case, the delta V is given as a function of inclination. In the second case, delta V is given as a function of ascending node.

DISCUSSION OF FIGURE 15

- (1) It is seen that the delta V requirements for out-of-plane orbit corrections are nearly 100 times the maximum delta V of in-plane corrections. Therefore, these results clearly predominate when considering mission feasibility.
- (2) When $I=0$, there is no distinction between ΔV for different values of Ω . This is because Ω becomes undefined. (Figure 15(A))
- (3) In Figure 15(A), the curve for $\Omega=0$ should intersect the abscissa at $I=7.3$ deg. It fails to do that because of the granularity of the data. Remember that for $I=7.3$ deg. and $\Omega=0$, the orbit plane is in equilibrium, and no delta V is required.

- (4) Inclined orbits require much more delta V than do the nominally equatorial case.
- (5) For a nominal inclination of 40 deg. (desirable for "Halo" orbits), the delta V requirements are nearly the same for all Ω . However these orbits require nearly the maximum delta V.

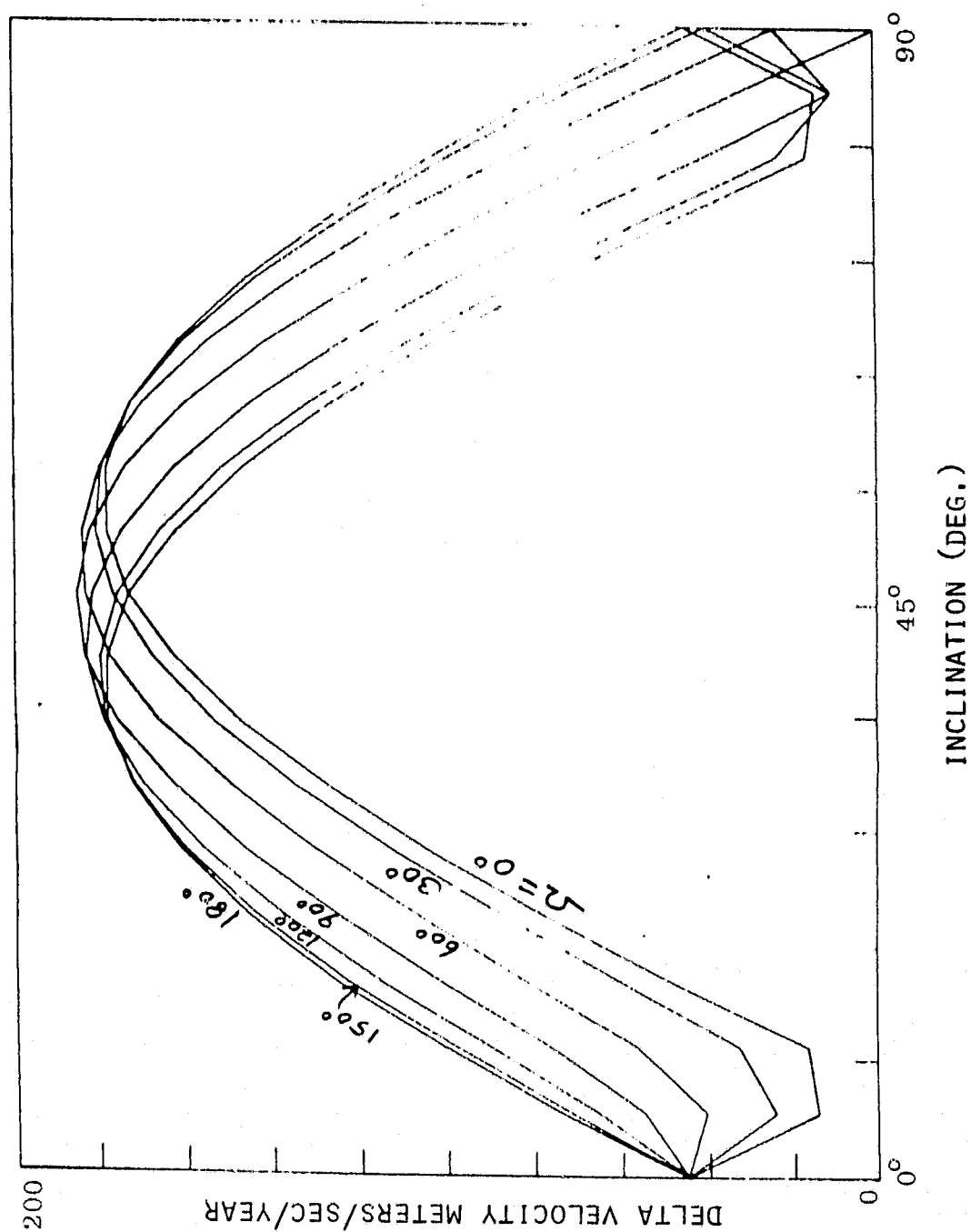


FIGURE 15: OUT-OF-PLANE VELOCITY CORRECTION
(A) DELTA-V VS INCLINATION

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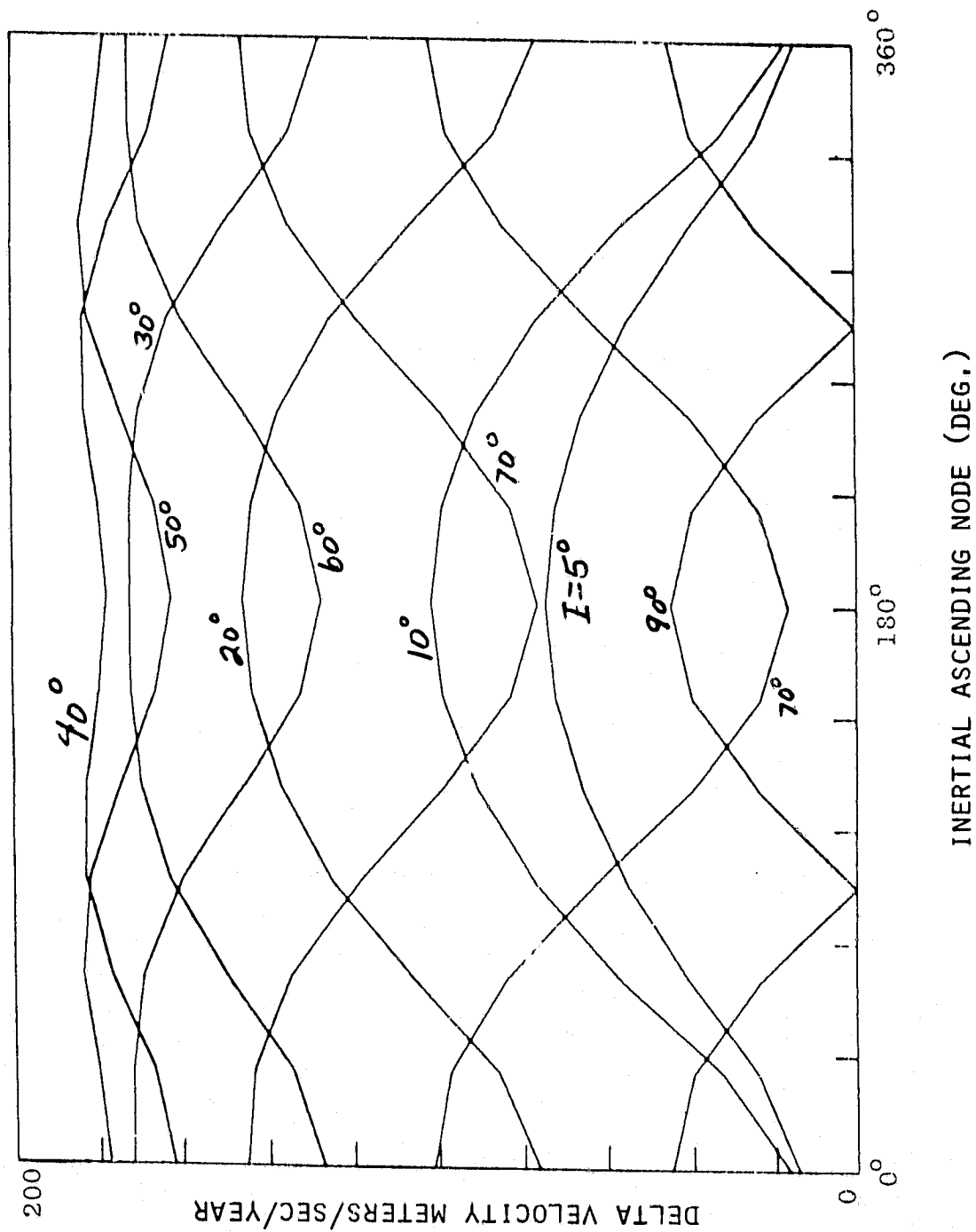


FIGURE 15: OUT-OF-PLANE VELOCITY CORRECTION
(B) DELTA V VS ASCENDING NODE

6.0 DISCUSSION OF MULTISATELLITE SYSTEMS

6.1 Problem Areas

It has been proposed that Inclined Circular Geosynchronous (Figure 8) orbits may be useful as a means to significantly increase the number of 24-hr. satellites in space. Several authors have investigated the satellite-ground station geometry for such orbits.

This report has been concerned with the orbital mechanics of inclined, circular geosynchronous orbits. It was found that station-keeping velocity requirements could be an order of magnitude larger than for equatorial orbits.

Whereas previous studies on this topic have been concerned with the utility of a single satellite in a nonstationary geosynchronous orbit, additional studies need to be concerned with a system of such satellites. The emphasis should be on mission design concepts that illustrate the practicality of this approach.

We suggest an extension of the work on applications of nonstationary geosynchronous (24-hr.) orbits, with the following primary objectives:

- (1) Study the applications of geosynchronous orbits that have arbitrary eccentricity and inclination.
- (2) Develop mission design concepts for multi-satellite "constellations" in geosynchronous orbits.

6.2 Mission Design Concepts

An appropriate next step in this study would be to develop mission design concepts for a "constellation" of satellites. The followings are some suggested study areas:

- (1) Develop mission design objectives and groundrules for a constellation of satellites in geosynchronous orbits. Take into consideration:
 - Individual user requirements,
 - Ground station tracking requirements,
 - Launch vehicle capabilities.
- (2) Determine the number of satellites required in order that a given ground location has continuous coverage. Consideration must be given to optimum eccentricity and inclination of the orbit. This part of the mission design should consider the longitude and latitude of the ground stations, as well as the equatorial crossing longitude of the satellites.
- (3) Develop an "inclination-offset" technique that minimizes out-of-plane orbit correction requirements. This concept has been used with existing equatorial satellites, and must be generalized to the case of inclined orbits. There is an additional degree of freedom to this problem since each satellite in a system has a different equatorial crossing point relative to inertial space.
- (4) Study the relative motion between satellites and determine the delta V requirements to maintain desired orbit positions.
- (5) Investigate the problem of collision and interference avoidance at the equatorial crossings.

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